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SUNSPOT ANALYSIS AND PREDICTION

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I. INTRODUCTION

Extensive literature is available on solar activity of a cyclic nature, particularly concerning the solar phenomena commonly called sunspot activity. The extent of this literature can best be emphasized by pointing out that the bibliography of solar activity compiled by Eckstein(1) contains 164 pages. On the other hand, the literature pertaining to solar cycle prediction is not nearly as extensive. In this more selective field both Scissum(2) and Vitinskii(3) present excellent discussions and surveys of solar cycle prediction models.

For some time the scientific community has accepted a definite correlation between sunspot activity and aurora intensity; and recent discoveries have revealed that a definite correlation also exists between the density of the upper atmosphere and solar activity of the sunspot nature. The path of earth satellites in orbit and their orbital lifetime depend on the upper atmosphere density. As a result, future plans for space research using orbiting satellites depend on an accurate forecasting of the variation of the upper atmosphere density resulting from the cyclic nature of sunspot activity. This is particularly true for planned vehicles such as the SkyLab and the Space Shuttle, along with the sophisticated laboratory modules the latter will deliver into orbit.

Sunspot activity is commonly measured according to an index called the Wolf number, and the raw data is presented in terms of monthly values for the Wolf number. This index, based on spot number and relative size, has been observed almost continuously in Zurich, Switzerland since the middle of the eighteenth century. In connection with empiricostatistical methods of analysis and prediction, the monthly values of the Wolf number are generally smoothed over a yearly period using statistical averaging procedures. An example of the smoothing effect on an eleven year cycle is shown in Figure 6.

Tabulations of Wolf numbers can be found in Vitinskii. Additional information concerning sunspot activity based primarily on aurora intensity is available for a much longer period of time. Schove[4] has compiled and evaluated this data going back to early Roman history.

The references listed previously provide an excellent summary of prediction procedures used and their results. However, these procedures and results are now inadequate for the prediction accuracy required in forecasting the density of the upper atmosphere when calculating the path and lifetime of satellites. Currently the Wolf number prediction activities fall into three types of procedures.

a. Identification of parameters combined with regression analysis.

b. Superposition methods.

c. Attempts at causal postulations.

These procedures are all reviewed concisely by Scissum; however, the binary star hypothesis postulated by Huntington[5] is not mentioned in this survey. It should be mentioned that Huntington's causal hypothesis and prediction efforts are discussed in the progress report for Contract NAS8-21179, dated February 1, 1968**.

The lack of accuracy and reliability of the many varied current sunspot prediction procedures resulted in the award of Contract NAS8-21179 by the Marshall Space Flight Center to the Graduate Institute of Technology of the University of Arkansas. The scope and effort of this contract were to consist of attempts to develop a more reliable prediction procedure for sunspot activity for at least one future eleven year period commonly designated as an eleven year cycle. This report is the culmination of these efforts.

The lack of reliability of present predicition procedures based primarily on short range data (1750 AD to 1970 AD) convinced the author than an approach

**See Appendix D

was needed that looked at the sunspot predicition procedure from a long-range viewpoint such as that initiated by Schove in his classical review of cyclic solar phenomena. The ideal approach would be to develop a theory based on accurate postulation of the causative factors involved in the formation of sunspots. Since this ideal may not be realized in the foreseeable future, the next best alternative involves an accurate functional representation of all existing sunspot data, both quantatative and qualitative -- ancient and modern.

After evaluating several procedures applying rarely used and exotic mathematical functions and procedures, the author finally concluded that a unique application of common trignometric functions appeared to yield fruitful results in the development of a mathematical model capable of describing all the available sunspot data. The development of the somewhat intuitive procedure that is described in this report is the result of many trials over the period of four years. The numerous trials will not be described; only the significant assumptions and results are presented.

It must be emphasized that this report presents only the foundation for an analytical analysis of existing data. The results given in Table 1, on page 20, and in Figure 1 show numerical predictions of Wolf numbers for the next sixty years. However, it must be pointed out emphatically that significant refinement is still needed, since the mathematical model for several periods in the past exhibits discrepancies which are readily noticed. But, it is anticipated that the procedure described in this report can be easily modified and expanded in such a manner that it accurately represents not only modern data (1750 AD to 1970 AD) but also the ancient data compiled by Schove that goes back almost continuously to 200 BC. The author is of the opinion that if the procedure developed in this report does not produce reliable predictions, then only an accurate causative theory will finally solve the solar cycle prediction problem.

II. BASIC ASSUMPTIONS

The basic assumptions and procedural guidelines are summarized and discussed below:

A. All existing sunspot data, both quantitative and qualitative, ancient and modern, is taken into consideration in the analysis. Information presented by Schove is accepted as being the best presently available concerning long range sunspot activity. At the present state of the art further smoothing of modern data (1750 AD to 1970 AD) will not decrease the accuracy of the predictions.

In order to present a graph of sunspot activity that appears to be amenable to mathematical analysis, further smoothing was found desirable in connection with the twelve (or thirteen) month smoothed data commonly presented in the literature such as Table 2 in Vitinskii. For use with the analysis of this report the monthly Wolf number data were smoothed by calculating an average Wolf number for a sequence of thirty-one months, and assigning this average value to the middle month or the sixteenth month in the sequence. The effect of this longer sequence smoothing is shown in Figure 6. In connection with this procedure the smoothed annual Wolf number is commonly defined as the average of the smoothed monthly values for the designated calendar year. Such average annual Wolf numbers are also shown in Table 2 of Vitinskii.

In order to use the information presented by Schove it was necessary to have numerical values for the data that he presents. This was accomplished by assigning numerical values to the maxima, graphing the maxima and minima, and drawing typical curves between the resulting points. The intermediate values were then determined from this graph. Yearly Wolf numbers obtained from the thirty-one month smoothing procedure, and the annual Wolf numbers based on the Schove data are presented in Appendix A. A condensed plot of this data is shown in Figure 7. For convenience in applying functional analysis, alternating signs were used for adjacent cycles in the tabulation of the annual Wolf numbers shown in Appendix A.

This procedure is similar to that used by Jose[6]. The resulting effect resembles a skewed sine or cosine curve as shown by the actual data curve in Figure 5.

A gap exists for the Schove data for the third century AD. To complete these data continuously back to 220 BC, the unknown peaks in the Schove data for the third century AD were assumed small and similar to the low values of the maxima in the seventeenth century. Certain maxima values are also indicated in various sections of this report for years prior to 220 BC; however, these values were not used in the establishment of the prediction functions presented in this report.

B. Groups of eleven-year sunspot cycles with maximum Wolf numbers greater than 100 have long-range periods greater than 200 years. Groups of sunspot cycles with intermediate (70 to 100) and small (40 to 70) Wolf number maxima <u>may</u> have long-range periods less than 200 years.

The conclusions concerning the long-range periodicity assumptions were derived based on the following procedure. An eleven-year sunspot maxima identification and coding system was established beginning with the present cycle (peak at 1969) and going backward in time as far as the Schove data are available. This identification and coding system is shown in Table 2 on page 21. This table shows the year of the maximum annual value of the Wolf number during its eleven-year cycle along with the maximum value of the Wolf number as given by Appendix A. This table also shows the code number associated with the year of maximum activity in the eleven-year cycle. This code number is introduced to assist in the identification of long-range cycles. In order to fill in the gaps occurring in the Schove data, the years shown in parenthesis in Table 2, page 21, are assumed to be the years of maximum activity in the eleven-year cycle.

Since three or four large maxima associated with the eleven-year cycle usually occur consecutively, long-range periodicities were sought that included four consecutive eleven-year cycles as one group. Such groups are readily identified in Figure 7. In Figure 7, examples of such groups are the periods 1934 to 1971, 1834 to 1879, 1712 to 1756, 1543 to 1587, 1115 to 1155, 707 AD to 749 AD, and 86 BC to 129 BC. Long-range periodicities were sought among such groups with large maxima so that an orderly and logical procedure for building a mathematical model could be developed that would eventually lead to reliable prediction functions.

An example of this search for long-range periodicities is shown in Table 3 on page 24. In this search all Wolf numbers are assumed to be positive. Periodicities for groups shown in Table 3 are determined by an indication of almost equal values for the four peak totals for adjacent groups or for groups occurring at regular intervals. The time interval for the long-range periodicities in Table 3 is then determined by the number of eleven-year cycles that occur between corresponding maxima in adjacent groups or groups that occur at regular intervals that exhibit such periodicities.

In Table 3 on page 24 an example of a long-range periodicity and a non-periodicity is given by the column for the 18 and 36 eleven-year cycle interval. In this column totals for adjacent groups do not show the equality property. This is particularly true for the first and second groups which have a total of 555 and 375 respectively. On the other hand beginning with the first group, the groups that have a long period of 36 eleven-year cycles show consistently high totals of 555, 560, 460, 500, 470 and 540. For this reason, a long period of 18 eleven-year cycles (198 to 202 years) as postulated by Schove was rejected in favor of a longer period of 36 cycles (395 to 404 years) for the groups beginning with cycles 1 through 4 (1969, 1958, 1948, 1938).

Other suggested periodicities such as the 7 cycle (78 year), 8 cycle (89 year), and 16 cycle (177 year) also do not show up in the tabular listing in Table 3 on page 24. These periodicities should show up in the 28 and 32 peak intervals, but they do not. The same procedure was used for other groups of four cycles in the modern era. The groups having periodicities considered significant are shown in Table 4 on page 27.

An extremely important conclusion is drawn from the results of Table 3 and Assumption B. The three periods of high sunspot activity during the past three centuries (1935 - 1970, 1835 - 1870, 1755 - 1798) are three independent populations with periods close to 400 years, 500 years, and 610 years respectively. As a result these populations do not play a significant part in the prediction of sunspot activity for the next 50 years. This unusual conclusion will be discussed in detail in section V on page 16 of this report.

C. A set of functions that describes sunspot activity needs to provide for both amplitude modulation and frequency modulation.

A perusal of the modern data shown in Figure 1 indicates what is commonly called the beat phenomena or amplitude modulation in vibration analysis. The frequency modulation of sunspot data was indicated more than forty years ago by Yule[7] by his general conclusion that sunspot numbers behaved in a manner similar to data that would be given by observations of disturbed periodic movements. As an example of the disturbed periodic characteristic of the sunspot numbers, the beat period for the 36 cycle periodicity varies from 395 to 404 years.

The major effort of this report is based on an analysis of the amplitude modulation characteristics of the sunspot data. This effort is explained in detail in the next section. A perturbation procedure that introduces a slight variation in the beat periods of the functions used is discussed in section IV. The predictions shown in Figure 1 do not include any frequency modulation correction.

D. The extremely high maximum of 1370 to 1374 must be included in any analysis of sunspot activity.

According to the Schove data the maximum occurring during the period 1370 to 1374 is the only one that approaches the modern peak value occurring in the period 1957 to 1959. Because of this, one would think that the earlier extremely large maximum should be considered in connection with a possible 53 cycle (580 to 590 years) periodicity, as shown in Table 3 on page 26, which would coincide with the modern high intensity decades. However, in the preliminary prediction calculations of this report, a 58 cycle (535 to 545 years) periodicity was chosen instead to prevent doubling up with other functions and to provide for the 78 to 81 year cyclic behavior of large maxima mentioned by many authors. The 58 cycle periodicity will be discussed in detail in the next section.

E. Functions are used that describe accurately several groupings of adjacent sunspot cycles (three to five), with a period greater than 200 years, but have small values for the remainder of the period. Emphasis is placed on fitting the cycles with large maxima.

The next section is devoted to the discussion of the derivation and development of the amplitude modulated functions that give the predictions shown in Figure 1. In section IV frequency modulation adjustments are derived in the form of perturbation functions for the long periods of the amplitude modulated functions.

III. DEVELOPMENT OF AMPLITUDE MODULATION FUNCTIONS

In Assumption C it is stated that amplitude modulation is indicated in the sunspot behavior. The development of the functions used to describe the complex beat phenomena will now be illustrated by using as an example the following simple function.

$$W = M \left(\mathrm{Cos}\,\frac{36\pi}{400}\,t + \mathrm{Cos}\,\frac{38\pi}{400}\,t \right)$$

where W represents the annual Wolf number, M is a scale factor, t represents time in years, and 400 is the beat period in years. This function has a positive maximum at t = 0 and t = 400 and at t equal to multiples of 400. A similar function is

$$W = M \left(\mathrm{Cos}\,\frac{35\pi}{400}\,t + \mathrm{Cos}\,\frac{37\pi}{400}\,t \right)$$

This function has positive maximum values at t = 0 and even multiples of 400, and has negative maximum values at t = 400 and odd multiples of 400. These functions are shown in Figure 2.

The expression for W can include more terms as shown by the following form

$$W = \frac{M}{n} \left[\mathrm{Cos}\,\frac{m\pi}{T}\,t + \mathrm{Cos}\,\frac{(m+2)\pi}{T}\,t + \ldots + \mathrm{Cos}\,\frac{(m+2n-2)\pi}{T}\,t \right]$$

where T is the beat period. Examples of these functions are shown in Figures 3a and 3b. The envelopes for these functions are not shown.

$T = 400 \quad M = 36 \quad n = 8$ and 24

$T = 400 \quad M = 35 \quad n = 8$ and 24

The sunspot data are not symmetrical with respect to a peak; therefore, the trignometric functions used must be multiplied by another function to describe the skewness of the sunspot data. An example of such a function is

$$W = M \left(a + b\,\mathrm{Sin}\,\frac{2\pi}{T}\,t \right) \left(\mathrm{Sin}\,\frac{\pi}{T}\,t \right)$$

or

$$W = M \left(a + b\,\mathrm{Sin}\,\frac{2\pi}{T}\,t \right) \left(\mathrm{Cos}\,\frac{\pi}{T}\,t \right)$$

In Figure 4a the scaled value of the sine function is shown in relation to the cycle whose peak occurs at 1948. The effect of the skewness function is shown for a = 2 and b = 1. In Figure 4b the scaled value of the sine function is shown in relation to the cycle whose peak occurs at 1788. This cycle is an example of extreme skewness; however, it is possible to approximate this skewness by using the function

$$W = M \left(2 + \text{Sin} \frac{2\pi}{T} t + 1/2 \; \text{Sin} \frac{3\pi}{T} t \right) \left(\text{Sin} \frac{\pi}{T} t \right)$$

A use of the previous equation is shown in Figure 4b. This figure shows the thirty-one month smoothed sunspot data and the following function for the annual Wolf number.

$$W = W_1 = \frac{77}{24} \left[2 + \text{Sin} \frac{80 \pi}{399} t \right] \left[\sum_{n=1}^{24} \text{Cos} \frac{(12 + 2n) \pi}{399} t \right]$$

where the sine term is set equal to zero at 1957.

Figure 5 shows that one W function is not sufficient to describe accurately a group of peaks such as the (1969, 1958, 1948, 1938) grouping. Therefore, another function is introduced in the following manner.

$$W = W_1 + W_2$$

where

$$W_1 = \frac{45}{24} \left[2 + \text{Sin} \frac{80 \pi}{399} t \right] \left[\sum_{n=1}^{24} \text{Cos} \frac{(12 + 2n) \pi}{399} t \right]$$

$$W_2 = \frac{38}{8} \left[-\left(2 + \text{Sin} \frac{80 \pi}{399} t\right) \right] \left[\sum_{n=1}^{8} \text{Cos} \frac{(30 + 2n) \pi}{399} t \right]$$

and the sine term is set equal to zero at 1957 for W_1 and 1947 for W_2.

This procedure was continued and functions were introduced to describe the three periods (1843 - 1878, 1766 - 1799, 1712 - 1745), so that the composite function is given by $W = W_1 + W_2 + W_3 + W_4 + W_5$ where W_1 and W_2 have been given previously and

$$W_3 = \frac{45}{8} \left[-\left(2 + \text{Sin} \frac{94 \pi}{500} t\right) \right] \left[\sum_{n=1}^{8} \text{Cos} \frac{(37 + 2n) \pi}{500} t \right]$$

$$W_4 = \frac{40}{20} \left[2 + \text{Sin} \frac{122 \pi}{607} t \right] \left[\sum_{n=1}^{20} \text{Cos} \frac{(38 + 2n) \pi}{607} t \right]$$

$$W_5 = \frac{45}{12} [-(2 + \mathrm{Sin} \frac{110\ \pi}{610} t)][\sum_{n=1}^{12} \mathrm{Cos} \frac{(41 + 2n)\ \pi}{610} t]$$

where the sine term is set equal to zero at 1862 for W_3, at 1780 for W_4, and 1729 for W_5. Table 5 on page 29 shows the value of the sum of the five functions for 1750 to 2150.

As Table 5 shows, the five combined functions show only small values for the years 1970 to 2100. This is the basis for the conclusion mentioned previously that the three groupings (1935 - 1970, 1835 - 1870, 1755 - 1798) were independent populations. In addition, the grouping (1712 - 1745) also appears to be another independent population.

Thus, it was concluded that significant prediction for the next fifty or sixty years is dependent on populations occurring before 1700 and on populations with low values for the maxima. Two additional populations are analyzed and used in the preliminary prediction of this report as shown in Figure 1.

A grouping of maxima of medium values occurs in the neighborhood of 1625 (1610 - 1645). This grouping was selected because only two cycles were repeated (133 at 512 AD and 134 at 501 AD) when compared with the five functions previously used. The coded data for this grouping is shown below in chronological order with the cycles that this grouping predicts.

Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number
167	130	70	133	512	130	99	887	90
168	118	70	134	501	150	100	872	130
169	105	110	135	490	90	101	862	90
170	96	70	136	479	110	102	850	110
		320			480			420
65	1259	90	31	1639	70	-3	2014	...
66	1249	70	32	1626	100	-2	2003	...
67	1239	90	33	1615	90	-1	1991	...
68	1228	90	34	1604	80	0	1979	...
		340			340			

In accordance with Assumption C, any prediction should include the unusually large maximum occurring around 1372. The function total for the five functions previously introduced accounts for only one-half of the peak value that is usually attributed to the 1372 maximum. Another grouping that includes this peak is shown below in chronological order along with the cycles that it influences in the prediction.

Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number
171	86	70	113	735	60	55	1372	190
172	76	70	114	724	130	56	1362	150
173	66	125	115	714	130	57	1353	70
174	53	130	116	699	60	58	1336	60
		395			380			470
-3	2014	...						
-2	2003	...						
-1	1991	...						
0	1279	...						

The composite prediction curve for seven functions, shown in Figure 1, is obtained by adding the values for the following two functions to the sum of the five functions given in the previous table.

$$W_6 = \frac{40}{12} [\ 2 + \text{Sin}\ \frac{64\ \pi}{376}\ t\ ][\ \sum_{n=1}^{12}\ \text{Cos}\ \frac{(18 + 2n)\ \pi}{376}\ t\ ]$$

$$W_7 = \frac{35}{32} [\ -(2 + \text{Sin}\ \frac{116\ \pi}{641}\ t)\ ][\ \sum_{n=1}^{32}\ \text{Cos}\ \frac{(26 + 2n)\ \pi}{641}\ t\ ]$$

where the sine term is zero at 1625 for W_6, and at 1371 for W_7.

For handy reference the predicted Wolf numbers, shown in Figure 1, are given in Table 1 on page 20. Table 6 on page 32 contains the sum of the seven functions for the period (1350 - 1394) to show that the functions used do account for the 1372 extra large maximum. The Fortan IV program that was used in the calculations is listed in Appendix C.

A comparison of Table 2 with Appendix C shows that the fit is good for the magnitude of the large maxima (120 to 190); but the phase relationship between the calculated and the actual values leaves something to be desired for these maxima. In the calculations of this section the long period (T) is assumed constant and an average value was used for each of the seven functions. In the next section a procedure is discussed that introduces a slight perturbation in the value of the long period (T) with respect to time.

A comment is also needed at this time concerning the procedure to determine the actual functions used and the individual parameters associated with these functions. The peak or maximum values were chosen by visual comparison with the actual data for the years for which these data are available. The periods of the functions used at their maximum value is approximately equal to the period of the middle peak of the grouping. Again this was determined by visual inspection. The double angle sine term that appears to give the best fit turns out to be the period of the chosen functions at their largest value. A least squares fit was attempted but proved too lengthy in time (personnel and computer) considering the scope of this effort. As stated before, further feasible extensions of these efforts concerning the fit of the functions with experimental data is discussed in section V.

IV. FREQUENCY MODULATION ADJUSTMENTS

An inspection of Figure 1 shows that for the period of 1880 to 1905 the calculated peak values are from three to five years out of phase with the data. Adjustments for these discrepancies can be made, particularly at large maxima, by introducing slight perturbations in the long periods by making the long period a function of time. That is: $T = T(t)$. The following simple expression was chosen to achieve the slight temporal perturbations required.

$$T(t) = T_o + A \operatorname{Sin} \alpha t + B \operatorname{Cos} \beta t$$

Five equations are needed to determine the five parameters A, B, T_o, α, and β. The five following expressions are given as an example.

$$407 = T_o + A \operatorname{Sin} 407\alpha + B \operatorname{Cos} 407\beta$$

$$404 = T_o + A \operatorname{Sin} 808\alpha + B \operatorname{Cos} 808\beta$$

$$401 = T_o + A \operatorname{Sin} 1203\alpha + B \operatorname{Cos} 1203\beta$$

$$400 = T_o + A \operatorname{Sin} 1600\alpha + B \operatorname{Cos} 1600\beta$$

$$400 = T_o + A \operatorname{Sin} 2000\alpha + B \operatorname{Cos} 2000\beta$$

In the preceding equations the origin of the time variable was chosen at 53 BC. The equations just given are transcendental with an infinite number of roots. The roots were chosen so that the perturbations were small and T_o approximately equal to the average long period for several groupings of large maxima. Shown in Table 7 on page 33 are the values of the parameters A, B, T_o, α, and β that are introduced to achieve a frequency modulation adjustment of the seven Wolf number functions given in Table 1 on page 20. The values of these parameters for five additional functions now being considered (W_8, W_9, W_{10}, W_{11}, W_{12}) are also shown in Table 7.

A modification of function W_2 shown above will be used to illustrate the need for and the effect of a variable long period T. In Table 8 on page 34 are shown the years of large Wolf number maxima for which W_2 has significant values.

Column 2 shows the actual or Schove data peak years; while column 3 shows the comparable years of calculated maxima for W_2 with a constant T. Column 4 shows the comparable years of calculated maxima for W_2 with a variable T; and column 5 shows the value of T calculated by the following expression.

$$T = 402.138 + 5.002 \text{ Sin } .005297t - 6.518 \text{ Cos } .004121t$$

The values of column 3 indicate that W_2 with a constant T (T = 400) shows a good agreement with the modern peaks (1 - 5 and 37 - 41) as expected; but with the two earlier groups of peaks (73 - 77 and 109 - 113) the agreement is poor. In fact, the displacements of 4 to 8 years for the calculated peaks, as compared to the actual, for the latter two groups corresponds to a phase shift of one-half of the eleven-year cycle. This may explain why such common superposition methods as periodogram analysis and harmonic analysis have not achieved any marked degree of success. The phase shifts are of sufficient magnitude to cause a cancellation of effects in the summations or integrations used in the commonly applied superposition procedures.

In contrast, good agreement is achieved for these two groups (73 - 77 and 109 - 113) when a variable T is used as indicated. None of the displacements of the calculated peaks exceeds three years. The refinement of fit possible for long-range data using a variable T is beyond the scope of this report; and further conclusions and recommendations in this area of investigation will be discussed in the next section.

V. CONCLUSIONS AND RECOMMENDATIONS

As mentioned on page 7 at the end of the discussion on Assumption B, the extremely important conclusion is reached in this report that the three periods of high sunspot activity during the past three centuries (1935 - 1970, 1835 - 1870, 1755 - 1790) are three independent populations. And the conclusion is also reached after an examination of the Schove data that these three populations have long periods of approximately 400 years, 500 years, and 610 years respectively with the result that these populations do not play a significant part in the predicition of sunspot activity for the next 50 years.

This conclusion differs from many other conclusions which postulate such periodicities as 80 year, 100 year, 178 year and others of similar magnitude. The most frequently discussed periodicity of 7 eleven-year cycles of approximately 80 years is based on a correlation with the planetary movement of Jupiter and Saturn as suggested by Jose or on a similar correlation with the periodicity of the nearest double star Alpha Centauri as postulated by Huntington. The result of such postulation is that emphasis is placed exclusively on fitting the smoothed modern Wolf number data (1750 - 1970) using mathematical or statistical analysis and then using any correlation obtained in the prediction of future cycles. The assumption just discussed automatically precludes the inclusion of long-range cycles with periods greater than 200 years. This assumption also implies that the causal nature of sunspot activity is exogenic with particular emphasis being placed on correlation with planetary motion. This implication results from the common misconception that correlation necessarily implies cause and effect. The assumption just mentioned also precludes the consideration that endogenic phenomena are responsible for sunspot activity. Considering the atomic fission and fusion activity of the total mass of such a large mass as the sun, it appears a reasonable assumption that the probability of endogenic behavior with periods longer than 200 years is greater that such periods in the 60 to 200 year range.

The previous discussion emphasizes the philosophy underlying this report which is stated in Assumption A; namely, that all existing sunspot data, both quantitative and qualitative, ancient and modern needs to be taken into consideration in the analysis. The goal expressed by this philosophy is an extremely ambitious one and this report is but a beginning of such an endeavor. Recommendations concerning the continuation of this effort will now be discussed.

Some of the possible extensions of this effort have already been mentioned in sections III and IV. Such efforts should achieve two things. First, the areas of insufficient agreement shown in Figure 1 such as 1790 to 1810, 1880 to 1900, and 1910 to 1930 periods need to be improved. Secondly, the agreement with the Schove data before 1750 needs to be adjusted so that calculated values for the years of maxima activity show less deviation both in magnitude and location.

The poor agreement for the periods just mentioned in the modern era can be improved by the addition of additional functions. However, the addition of functions considering only discrepancies in the modern data violates the fundamental premise on which the whole effort is based. General examples of additional functions (W_8, W_9, W_{10}, W_{11}, W_{12}) are shown in Table 7 on page 33.

Therefore, it is necessary to make the frequency modulation adjustments for the seven functions presently used as given in Table 1 on page 20 so that all the Schove data information is applied and accurately represented by the seven functions presently used before any further efforts are made to improve the modern data. An example of such adjustments for frequency modulation are shown in Table 7 on page 33.

In addition to the improvements just mentioned; some procedure, such as least squares, should be used to improve the curve fitting which up to now has been based on visual inspection. Most of the preliminary effort and trial calculations were performed on a Hewlett-Packard Model 9100 calculator, with some of the final calculations performed on a G. E. Mark II computer. To achieve an appreciation of the magnitude of the calculations based primarily on visual inspection and intuition that were performed, twelve years normal supply of printer paper for a Hewlett-Packard 9100 calculator was used in the preliminary calculations. This is an accurate indication of the amount of computer time required to continue and complete the investigation in the manner suggested at the beginning of this section.

The investigation is now at a stage where even the preliminary calculations must be performed on a large computer such as the G. E. Mark II, and further detailed investigation will require a considerable amount of computer time on a computer of the type as mentioned in the previous paragraph. In summary the following continuation of the procedure established in this report is recommended:

1. Introduction of frequency modulation adjustments to the functions presently used (W_1, W_2, W_3, W_4, W_5, W_6, W_7) and the use of a procedure such as least squares to determine the parameters in these functions with more accuracy.

2. Evaluate the results of the previous procedure and introduce additional functions where the fit is poor in both the modern and the ancient era.

3. Continue this procedure, adjusting parameters and adding functions, until a preassigned degree of accuracy has been achieved in comparison with the existing data.

The prediction curve resulting from this procedure should then be the best available and should be accepted with a fair degree of confidence by the scientific community.

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Table 1

Wolf Number Predictions for the Next Sixty Years Using the Functions Shown Below

Year (t)	Calc. Annual Wolf Number*	Year (t)	Calc. Annual Wolf Number*	Year (t)	Calc. Annual Wolf Number*
1971	-49	1991	-91	2011	-80
1972	-18	1992	-79	2012	-119
1973	- 1	1993	-54	2013	-152
1974	5	1994	-22	2014	-160
1975	10	1995	10	2015	-139
1976	19	1996	36	2016	-97
1977	31	1997	53	2017	-52
1978	39	1998	67	2018	-18
1979	37	1999	84	2019	3
1980	26	2000	109	2020	17
1981	9	2001	139	2021	32
1982	-6	2002	162	2022	49
1983	-17	2003	164	2023	64
1984	-21	2004	136	2024	71
1985	-24	2005	85	2025	69
1986	-28	2006	28	2026	60
1987	-39	2007	-15	2027	46
1988	-56	2008	-35	2028	30
1989	-74	2009	-42	2029	14
1990	-88	2010	-53	2030	0

$$W_1 = \frac{45}{24}\left[2 + \sin \frac{80\pi}{399} t \right]\left[\sum_{n=1}^{24} \cos \frac{(12 + 2n)\pi}{399} t \right]$$

$$W_2 = \frac{38}{8}\left[-(2 + \sin \frac{80\pi}{399} t) \right]\left[\sum_{n=1}^{8} \cos \frac{(30 + 2n)\pi}{399} t \right]$$

$$W_3 = \frac{45}{8}\left[-(2 + \sin \frac{94\pi}{500} t) \right]\left[\sum_{n=1}^{8} \cos \frac{(37 + 2n)\pi}{500} t \right]$$

$$W_4 = \frac{40}{20}\left[2 + \sin \frac{122\pi}{607} t \right]\left[\sum_{n=1}^{20} \cos \frac{(38 + 2n)\pi}{607} t \right]$$

$$W_5 = \frac{45}{12}\left[-(2 + \sin \frac{110\pi}{610} t) \right]\left[\sum_{n=1}^{12} \cos \frac{(41 + 2n)\pi}{610} t \right]$$

$$W_6 = \frac{40}{12}\left[2 + \sin \frac{64\pi}{376} t \right]\left[\sum_{n=1}^{12} \cos \frac{(18 + 2n)\pi}{376} t \right]$$

$$W_7 = \frac{35}{32}\left[-(2 + \sin \frac{116\pi}{641} t) \right]\left[\sum_{n=1}^{32} \cos \frac{(26 + 2n)\pi}{641} t \right]$$

*Calculated Wolf Number = $W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7$

Table 2

Identification and Coding System
For Eleven-year Maximum or Peak Values of
Annual Wolf Numbers

Peak Number	Peak Year A.D.	Annual Wolf Number	Peak Number	Peak Year A.D.	Annual Wolf Number	Peak Number	Peak Year A.D.	Annual Wolf Number
1	1969	-110	33	1615	- 90	65	1259	- 90
2	1958	190	34	1604	80	66	1249	70
3	1948	-145	35	1591	- 70	67	1239	- 90
4	1938	110	36	1581	130	68	1228	90
5	1928	- 75	37	1572	-150	69	1219	- 90
6	1918	85	38	1558	160	70	1202	150
7	1906	- 60	39	1548	-120	71	1193	- 90
8	1894	80	40	1539	130	72	1185	110
9	1883	- 65	41	1528	-150	73	1173	-110
10	1871	120	42	1519	80	74	1160	70
11	1860	- 90	43	1505	- 60	75	1151	-130
12	1849	115	44	1492	70	76	1138	150
13	1837	-130	45	1480	- 50	77	1129	-130
14	1829	65	46	1472	50	78	1118	150
15	1817	- 45	47	1461	- 70	79	1110	- 70
16	1804	50	48	1449	70	80	1098	150
17	1788	-130	49	1439	- 70	81	1089	- 85
18	1779	135	50	1429	70	82	1078	90
19	1772	-105	51	1413	- 50	83	1067	- 90
20	1761	70	52	1402	90	84	1052	50
21	1750	- 90	53	1391	- 90	85	1038	- 60
22	1739	110	54	1382	110	86	1027	90
23	1728	-140	55	1372	-190	87	1016	- 90
24	1718	130	56	1362	150	88	1003	130
25	1705	- 50	57	1353	- 70	89	994	- 60
26	1693	30	58	1336	60	90	986	90
27	1685	- 50	59	1324	- 90	91	974	-150
28	1675	60	60	1314	90	92	963	150
29	1660	- 50	61	1308	- 90	93	950	- 70
30	1649	40	62	1296	60	94	938	110
31	1639	- 70	63	1288	- 90	95	926	-150
32	1626	100	64	1276	90	96	917	90

Peak Number	Year A.D.	Annual Wolf Number
97	907	- 60
98	898	60
99	887	- 90
100	872	130
101	862	- 90
102	850	110
103	840	-150
104	829	130
105	821	- 60
106	809	130
107	798	-110
108	787	60
109	776	-110
110	765	150
111	754	- 90
112	745	150
113	735	- 60
114	724	130
115	714	-130
116	699	60
117	689	- 60
118	677	130
119	665	- 90
120	654	110
121	642	- 90
122	628	80
123	618	- 90
124	607	60
125	597	- 80
126	587	130
127	578	- 90
128	567	150
129	557	- 90
130	542	90
131	531	-130
132	522	90
133	512	-130
134	501	150
135	490	- 90
136	479	110

Peak Number	Year A.D.	Annual Wolf Number
137	465	- 60
138	452	130
139	441	-110
140	430	110
141	421	- 60
142	410	60
143	396	- 90
144	387	90
145	372	-150
146	362	130
147	354	-130
148	342	60
149	330	- 60
150	321	90
151	311	- 90
152	302	150
153	290	-110
154	277	60
155	265	- 50
156	252	60
157	240	- 60
158	230	60
159	219	- 50
160	208	90
161	196	-130
162	186	130
163	175	-110
164	163	70
165	152	- 70
166	141	70
167	130	- 70
168	118	70
169	105	-110
170	96	70
171	86	- 70
172	76	70
173	66	-125
174	53	130
175	42	- 70
176	31	70

Peak Number	Year A.D.	Annual Wolf Number
177	20	-130
178	8	70
	B.C.	
179	-4	- 70
180	-16	70
181	-27	-130
182	-42	110
183	-53	-150
184	-62	150
185	-72	- 70
186	-82	90
187	-91	-150
188	-104	130
189	-113	-130
190	-125	110
191	-135	- 90
192	-149	60
193	-163	-130
194	-172	70
195	-183	- 70
196	-192	130
197	-205	-130
198	-214	130
199	-233	- 90
200	-236	...
201	-249	...
202	-261	...
203	-272	...
204	-283	...
205	-293	...
206	(-304)	...
207	(-315)	...
208	-327	...
209	-339	-130
210	-349	130
211	-360	...
212	-271	...
213	(-382)	...
214	-393	130

Peak Number	Year B.C.	Annual Wolf Number
215	(-404)	...
216	(-415)	...
217	(-427)	...
218	(-438)	...
219	(-450)	...
220	-461	130
221	-471	-130
222	-481	130
223	-491	- 70
224	-501	130
225	-512	- 80
226	-522	130
227	(-533)	...
228	(-544)	...
229	(-555)	...
230	(-565)	...
231	(-575)	...
232	(-586)	...
233	(-597)	...
234	(-608)	...
235	(-618)	...
236	(-628)	...
237	(-638)	...
238	-648	150
239	(-658)	...
240	(-669)	...

Table 3

Examples of Search for Long-range Periodicities

Interval of 27* Eleven-year Cycles						Interval of 28* Eleven-year Cycles		
Code for Year of Maximum	Year of Maximum	Wolf** Number	Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number
1	1969	110	163	175	110	1	1969	110
2	1958	190	164	163	70	2	1958	190
3	1948	145	165	152	70	3	1948	145
4	1938	110	166	141	70	4	1938	110
	Total	555		Total	320		Total	555
28	1675	60	190	-125	110	29	1660	50
29	1660	50	191	-135	90	30	1649	40
30	1649	40	192	-149	60	31	1639	70
31	1639	70	193	-163	130	32	1626	100
	Total	220		Total	390		Total	260
55	1372	190	217	(-427)	...	57	1353	70
56	1362	150	218	(-438)	...	58	1336	60
57	1353	70	219	(-450)	...	59	1324	90
58	1336	60	220	(-461)	130	60	1314	90
	Total	470		Total	...		Total	310
82	1078	90				85	1038	60
83	1067	90				86	1027	90
84	1052	50				87	1016	90
85	1038	60				88	1003	130
	Total	290					Total	370
109	776	110				113	735	60
110	765	150				114	724	130
111	754	90				115	714	130
112	745	150				116	699	60
	Total	500					Total	380
136	479	110				141	421	60
137	465	60				142	410	60
138	452	130				143	396	90
139	441	110				144	387	90
	Total	410					Total	300

* This value indicates the number of eleven-year cycles that occur between corresponding maxima in adjacent groups.

**Only absolute values for the Wolf number were used in the search for long-range periodicities.

Interval of 28*
Eleven-year Cycles

Code for Year of Maximum	Year of Maximum	Wolf** Number
169	105	110
170	96	70
171	86	70
172	76	70
	Total	320
197	-205	130
198	-214	130
199	-233	90
200	-236	...
	Total	470?
225	-512	80
226	-522	130
227	(-533)	...
228	(-544)	...
	Total	?

Interval of 32*
Eleven-year Cycles

Code for Year of Maximum	Year of Maximum	Wolf Number
1	1969	110
2	1958	190
3	1948	145
4	1938	110
	Total	555
33	1615	90
34	1604	80
35	1591	70
36	1581	130
	Total	370
65	1259	90
66	1249	70
67	1239	90
68	1228	90
	Total	340
97	907	60
98	898	60
99	887	90
100	872	130
	Total	340
129	557	90
130	542	90
131	531	130
132	522	90
	Total	400
161	196	130
162	186	130
163	175	110
164	163	70
	Total	440
193	-163	130
194	-172	70
195	-183	70
196	-192	130
	Total	400

Code for Year of Maximum	Year of Maximum	Wolf Number
225	-512	80
226	-522	130
227	(-533)	...
228	(-544)	...
	Total	?

* This value indicates the number of eleven-year cycles that occur between corresponding maxima in adjacent groups.

**Only absolute values for the Wolf number were used in the search for long-range periodicities.

Interval of 18* and 36 Eleven-year Cycles

Code for Year of Maximum	Year of Maximum	Wolf** Number	Code for Year of Maximum	Year of Maximum	Wolf Number
1	1969	110	127	578	90
2	1958	190	128	567	150
3	1948	145	129	557	90
4	1938	110	130	542	90
	Total	555		Total	430
19	1772	105	145	372	150
20	1761	70	146	362	130
21	1750	90	147	354	130
22	1739	110	148	342	60
	Total	375		Total	470
37	1572	150	163	175	110
38	1558	160	164	163	70
39	1548	120	165	152	70
40	1539	130	166	141	70
	Total	560		Total	320
55	1372	190	181	-27	130
56	1362	150	182	-42	110
57	1353	70	183	-53	150
58	1336	60	184	-62	150
	Total	470		Total	540
73	1173	110	199	-233	90
74	1160	70	200	-236	...
75	1151	130	201	-249	...
76	1138	150	202	-261	...
	Total	460		Total	?
91	974	150	216	(-415)	...
92	963	150	217	(-427)	...
93	950	70	218	(-438)	...
94	938	110	219	(-450)	...
	Total	480		Total	?
109	776	110	235	(-618)	...
110	765	150	236	(-628)	...
111	754	90	237	(-638)	...
112	745	150	238	-648	150
	Total	500		Total	?

Interval of 53* Eleven-year Cycles

Code for Year of Maximum	Year of Maximum	Wolf Number
1	1969	110
2	1958	190
3	1948	145
4	1938	110
	Total	555
54	1382	110
55	1372	190
56	1362	150
57	1353	70
	Total	520
107	798	110
108	787	60
109	776	110
110	765	150
	Total	430
160	208	90
161	196	130
162	186	130
163	175	110
	Total	460
213	(-382)	...
214	-393	130
215	(-404)	...
216	(-415)	...
	Total	?

* This value indicates the number of eleven-year cycles that occur between corresponding maxima in adjacent groups.

**Only absolute values for the Wolf number were used in the search for long-range periodicities.

Table 4

Groups with Significant Long-Range Periodicities

Maxima at 1788, 1779, 1770, and 1761

Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number
17*	1788	130	72*	1185	110	127	578	90
18	1779	135	73	1173	110	128	567	150
19	1772	105	74	1160	70	129	557	90
20	1761	70	75	1151	130	130	542	90
		440			420			420

Code for Year of Maximum	Year of Maximum	Wolf Number
182	-42	110
183	-53	150
184	-62	150
185	-72	70
		480

*Interval of 55 Eleven-year Cycles

Maxima at 1750, 1739, 1728, and 1718

Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number
21*	1750	90	76*	1138	150	131	531	130
22	1739	110	77	1129	130	132	522	90
23	1728	140	78	1118	150	133	512	130
24	1718	130	79	1110	70	134	501	150
		470			500			500

Code for Year of Maximum	Year of Maximum	Wolf Number
186	-82	90
187	-91	150
188	-104	130
189	-113	130
		500

*Interval of 55 Eleven-year Cycles

Maxima at 1639, 1626, 1615, and 1604

Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number
31*	1639	70	65*	1259	90	99	887	90
32	1626	100	66	1249	70	100	872	130
33	1615	90	67	1239	90	101	862	90
34	1604	80	68	1228	90	102	850	110
		340			340			420

Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number
133	512	130	167	130	70
134	501	150	168	118	70
135	490	90	169	105	110
136	479	110	170	96	70
		480			320

*Interval of 34 Eleven-year Cycles

Maxima at 1372, 1362, 1353, and 1336

Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number
55	1372	190	113	735	60	171	86	70
56	1362	150	114	724	130	172	76	70
57	1353	70	115	714	130	173	66	125
58	1336	60	116	699	60	174	53	130
		470			380			400

Maxima at 1382, 1372, 1362, and 1353

Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number	Code for Year of Maximum	Year of Maximum	Wolf Number
54	1382	110	113	735	60	172	76	70
55	1372	190	114	724	130	173	65	130
56	1362	150	115	714	130	174	53	130
57	1353	70	116	699	60	175	42	70
		520			380			400

Table 5

Sum of Five Prediction Functions

Year	Calc. Wolf Number	Actual Wolf Number	Year	Calc. Wolf Number	Actual Wolf Number	Year	Calc. Wolf Number	Actual Wolf Number
1750	-85	-90	1790	-74	-105	1830	39	65
1751	-71	-75	1791	-51	-80	1831	15	60
1752	-43	-60	1792	-16	-65	1832	-3	40
1753	-13	-45	1793	15	-55	1833	-14	20
1754	6	-25	1794	34	-45	1834	-22	0
1755	12	-15	1795	39	-30	1835	-35	-40
1756	14	0	1796	40	-20	1836	-56	-100
1757	25	25	1797	42	-10	1837	-79	-130
1758	47	45	1798	45	-5	1838	-94	-120
1759	71	50	1799	45	0	1839	-89	-95
1760	83	60	1800	37	15	1840	-65	-70
1761	71	70	1801	22	30	1841	-31	-50
1762	41	70	1802	5	40	1842	0	-20
1763	8	60	1803	-8	45	1843	18	0
1764	-13	40	1804	-14	50	1844	26	15
1765	-21	25	1805	-15	45	1845	35	30
1766	-25	0	1806	-14	40	1846	53	55
1767	-37	-20	1807	-12	25	1847	81	80
1768	-60	-50	1808	-8	15	1848	106	110
1769	-83	-85	1809	-4	10	1849	115	115
1770	-88	-95	1810	2	5	1850	98	85
1771	-69	-100	1811	5	0	1851	63	65
1772	-32	-105	1812	4	-5	1852	22	60
1773	4	-60	1813	-1	-10	1853	-7	45
1774	25	-30	1814	-8	-15	1854	-23	30
1775	31	0	1815	-17	-25	1855	-31	15
1776	33	15	1816	-26	-40	1856	-44	0
1777	46	65	1817	-34	-45	1857	-66	-15
1778	70	115	1818	-37	-35	1858	-93	-45
1779	92	135	1819	-35	-30	1859	-110	-75
1780	94	105	1820	-27	-20	1860	-107	-90
1781	69	80	1821	-17	-10	1861	-82	-90
1782	27	55	1822	-6	-5	1862	-46	-70
1783	-12	30	1823	5	0	1863	-11	-60
1784	-35	0	1824	17	10	1864	13	-50
1785	-40	-20	1825	32	15	1865	26	-40
1786	-41	-65	1826	50	25	1866	36	-20
1787	-48	-100	1827	64	40	1867	53	0
1788	-63	-130	1828	69	60	1868	76	25
1789	-76	-125	1829	59	65	1869	100	60

Year	Calc. Wolf Number	Actual Wolf Number	Year	Calc. Wolf Number	Actual Wolf Number	Year	Calc. Wolf Number	Actual Wolf Number
1870	110	105	1920	27	55	1970	-81	-85
1871	101	120	1921	8	35	1971	-47	
1872	72	110	1922	-6	20	1972	-20	
1873	36	85	1923	-11	0	1973	-5	
1874	4	55	1924	-11	-10	1974	0	
1875	-17	30	1925	-14	-35	1975	4	
1876	-30	15	1926	-26	-60	1976	9	
1877	-45	10	1927	-44	-70	1977	16	
1878	-66	5	1928	-53	-75	1978	19	
1879	-92	0	1929	-60	-70	1979	16	
1880	-110	-20	1930	-47	-55	1980	10	
1881	-110	-45	1931	-28	-35	1981	4	
1882	-89	-55	1932	-13	-20	1982	-1	
1883	-54	-65	1933	-7	-10	1983	-4	
1884	-19	-60	1934	-6	0	1984	-5	
1885	7	-55	1935	-1	30	1985	-8	
1886	22	-35	1936	17	60	1986	-11	
1887	32	-20	1937	46	95	1987	-16	
1888	42	-10	1938	72	110	1988	-20	
1889	56	0	1939	80	100	1989	-22	
1890	69	10	1940	63	80	1990	-20	
1891	75	25	1941	31	60	1991	-14	
1892	68	60	1942	4	40	1992	-7	
1893	49	80	1943	-8	20	1993	0	
1894	23	80	1944	-10	0	1994	6	
1895	-1	70	1945	-19	-25	1995	12	
1896	-15	55	1946	-50	-65	1996	18	
1897	-19	40	1947	-98	-120	1997	26	
1898	-16	30	1948	-141	-145	1998	31	
1899	-14	20	1949	-149	-140	1999	32	
1900	-19	10	1950	-116	-105	2000	27	
1901	-29	0	1951	-58	-75	2001	18	
1902	-37	-5	1952	-6	-45	2002	10	
1903	-36	-15	1953	20	-25	2003	3	
1904	-25	-35	1954	27	0	2004	-2	
1905	-11	-50	1955	41	30	2005	-6	
1906	-2	-60	1956	79	95	2006	-9	
1907	-3	-60	1957	135	160	2007	-9	
1908	-10	-55	1958	181	190	2008	-6	
1909	-11	-45	1959	188	170	2009	-1	
1910	-3	-30	1960	148	130	2010	2	
1911	12	-15	1961	85	80	2011	3	
1912	26	-10	1962	28	50	2012	1	
1913	31	0	1963	-2	30	2013	-1	
1914	27	10	1964	-14	20	2014	-1	
1915	24	30	1965	-24	0	2015	0	
1916	27	60	1966	-48	-35	2016	-2	
1917	36	80	1967	-81	-75	2017	-6	
1918	44	85	1968	-104	-105	2018	-10	
1919	41	75	1969	-104	-110	2019	-11	

Year	Calc. Wolf Number	Actual Wolf Number
2020	-7	
2021	1	
2022	6	
2023	8	
2024	8	
2025	8	
2026	10	
2027	14	
2028	14	
2029	9	
2030	0	
2031	-8	
2032	-12	
2033	-12	
2034	-9	
2035	-8	
2036	-10	
2037	-13	
2038	-14	
2039	-13	
2040	-10	
2041	-7	
2042	-4	
2043	-3	
2044	-3	
2045	-3	
2046	-3	
2047	-3	
2048	-1	
2049	2	
2050	6	
2051	11	
2052	15	
2053	15	
2054	13	
2055	11	
2056	11	
2057	13	
2058	15	
2059	15	
2060	11	
2061	2	
2062	-9	
2063	-16	
2064	-18	
2065	-16	
2066	-14	
2067	-16	
2068	-19	
2069	-22	

Year	Calc. Wolf Number	Actual Wolf Number
2070	-19	
2071	-9	
2072	3	
2073	14	
2074	19	
2075	20	
2076	20	
2077	21	
2078	22	
2079	22	
2080	18	
2081	10	
2082	1	
2083	-7	
2084	-12	
2085	-13	
2086	-12	
2087	-11	
2088	-8	
2089	-6	
2090	-4	
2091	-3	
2092	0	
2093	2	
2094	2	
2095	0	
2096	-4	
2097	-9	
2098	-13	
2099	-14	
2100	-12	
2101	-9	
2102	-6	
2103	-4	
2104	-2	
2105	1	
2106	7	
2107	12	
2108	15	
2109	15	
2110	14	
2111	12	
2112	10	
2113	8	
2114	6	
2115	2	
2116	-2	
2117	-7	
2118	-12	
2119	-15	

Year	Calc. Wolf Number	Actual Wolf Number
2120	-16	
2121	-15	
2122	-12	
2123	-9	
2124	-6	
2125	-3	
2126	3	
2127	10	
2128	17	
2129	21	
2130	21	
2131	17	
2132	12	
2133	9	
2134	6	
2135	2	
2136	-3	
2137	-10	
2138	-15	
2139	-16	
2140	-12	
2141	-7	
2142	-3	
2143	-1	
2144	-1	
2145	1	
2146	4	
2147	9	
2148	12	
2149	11	

Table 6

Calculated and Actual Annual Wolf Numbers for the period (1350 AD - 1394 AD)

Year	Calc. Wolf Number	Actual Wolf Number	Year	Calc. Wolf Number	Actual Wolf Number	Year	Calc. Wolf Number	Actual Wolf Number
1350	-95	-40	1365	-12	85	1380	110	70
1351	-69	-50	1366	-39	45	1381	127	105
1352	-39	-65	1367	-64	30	1382	125	110
1353	-11	-70	1368	-97	0	1383	100	105
1354	10	-60	1369	-135	-50	1384	61	50
1355	28	-45	1370	-170	-110	1385	23	35
1356	50	-30	1371	-184	-180	1386	-4	0
1357	79	-15	1372	-169	-190	1387	-18	-20
1358	115	0	1373	-127	-180	1388	-28	-40
1359	148	40	1374	-73	-140	1389	-40	-60
1360	165	85	1375	-24	-110	1390	-56	-80
1361	157	130	1376	12	-70	1391	-68	-90
1362	123	150	1377	36	-35	1392	-66	-80
1363	74	145	1378	58	0	1393	-47	-60
1364	26	110	1379	83	35	1394	-18	-40

Table 7

Parameters in Frequency Modulation Adjustments

	W_1	W_2	W_3	W_4	W_5	W_6
Origin	42 B.C.	53 B.C.	135 B.C.	53 B.C.	104 B.C.	263 B.C.
T_o	402.229	402.138	497.93	615.094	612.916	379.719
A	-.79	5.002	.392	5.045	3.26	2.15
B	2.852	-6.518	-1.459	0.0	0.0	.672
α	.00583	.005297	.007343	.002912	.003085	.002876
β	.001514	.004121	.002099	.00	.00	.009055

	W_7	W_8	W_9	W_{10}	W_{11}	W_{12}
Origin	86 A.D.	28 B.C.	113 B.C.	53 A.D.	150 B.C.	103 B.C.
T_o	645.334	388.971	400.19	377.222	388.333	410.981
A	3.97	-.801	-2.992	-.342	-.576	1.734
B	0.0	-.802	-3.504	-.459	-.332	-1.037
α	.012704	.006747	.010379	.023376	.013493	.002528
β	.00	.004028	.004052	.00394	.005398	.005083

Table 8

Values of Variable T for Modification of Prediction Function W_2

Peak Number from Table 2	Actual or Schove Data Peak Year	Constant T T = 400 Peak Year	Variable T Peak Year	T
1	A.D. 1969	1968	1970	400.36
2	1958	1957	1959	400.18
3	1948	1947	1949	400.01
4	1938	1937	1938	399.87
5	1928	1927	1927	399.73
37	1572	1568	1568	399.85
38	1558	1557	1558	399.92
39	1548	1547	1548	399.99
40	1539	1537	1539	400.06
41	1528	1527	1529	400.13
73	1173	1168	1171	401.01
74	1160	1157	1162	401.00
75	1151	1147	1151	401.00
76	1138	1137	1141	401.00
77	1129	1127	1131	401.00
109	776	768	776	403.67
110	765	757	766	403.82
111	754	747	756	403.98
112	745	737	747	404.13
113	735	727	737	404.29
145	372	368	376	407.23
146	362	357	366	407.14
147	354	345	355	407.01
148	342	337	345	406.89
149	330	327	335	406.75
181	B.C. 27	32	32	396.20
182	42	43	42	395.92
183	53	53	52	395.62
184	62	63	62	395.39
185	72	73	71	395.16
217	(427)	432	429	397.43
218	(438)	443	439	397.85
219	(450)	453	450	398.25
220	461	463	460	398.66
221	471	473	471	399.12

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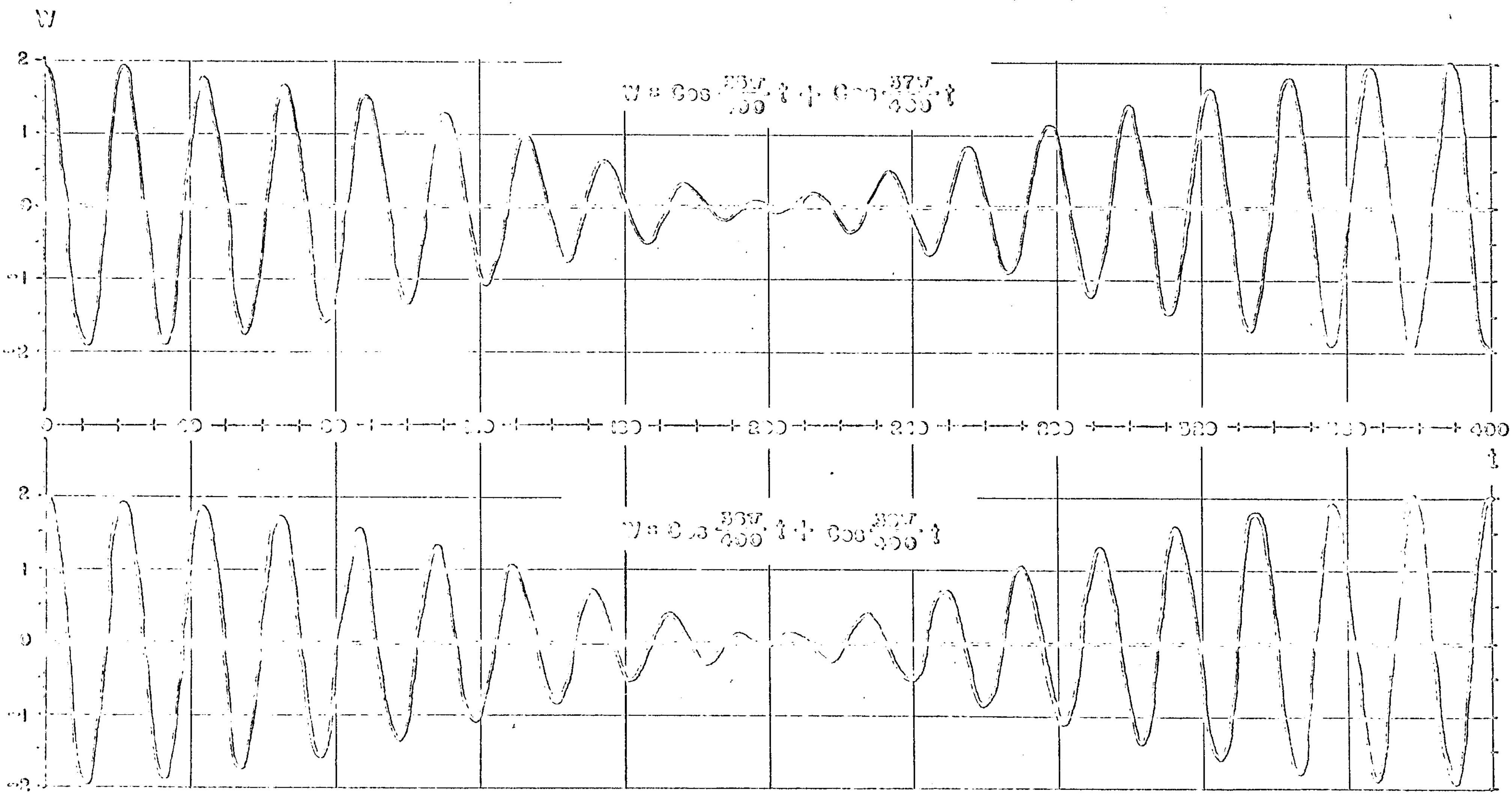

FIGURE 2 - EXAMPLE OF BEAT FUNCTION

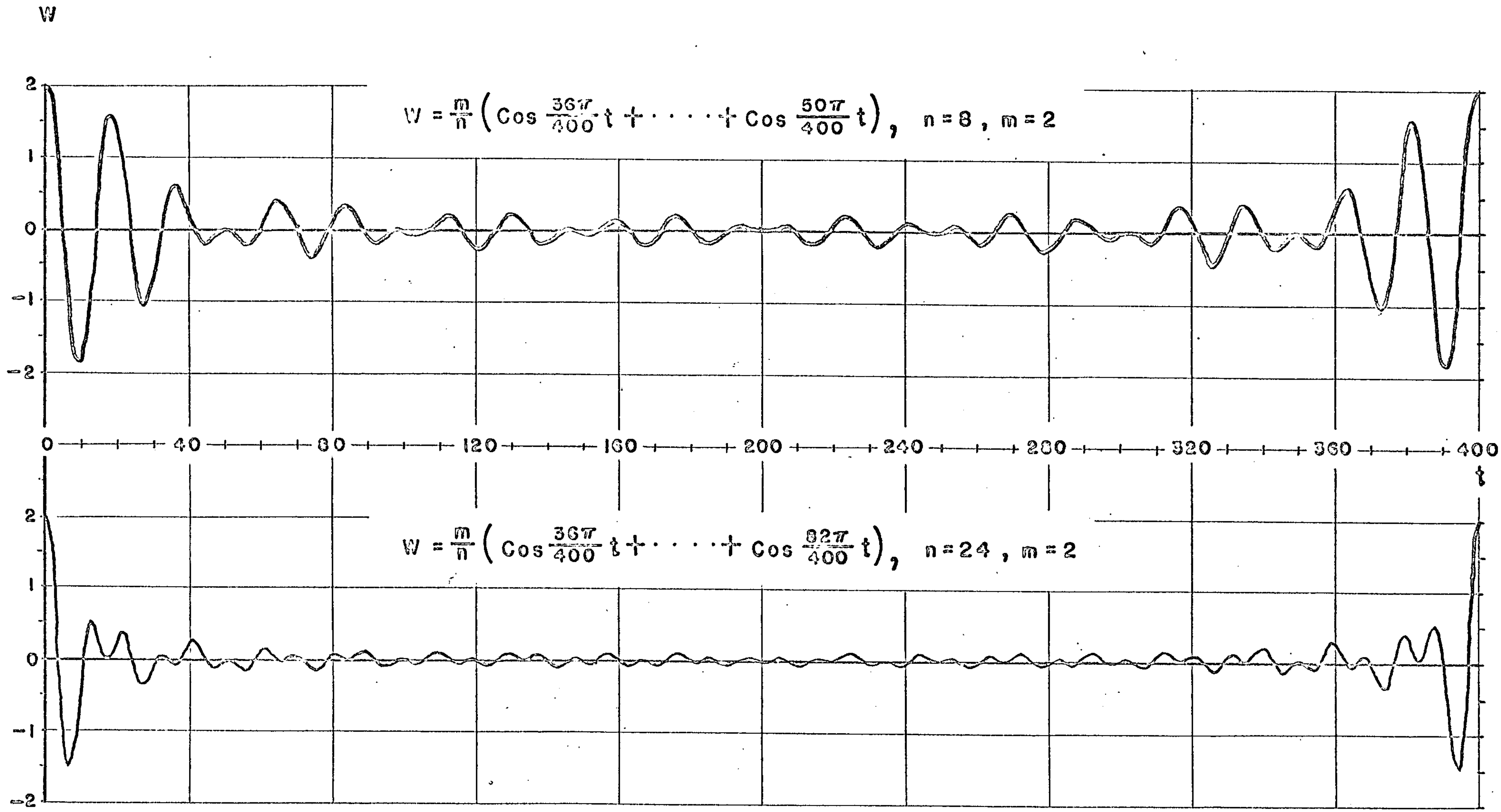

FIGURE 3a — EXAMPLES OF BEAT FUNCTIONS

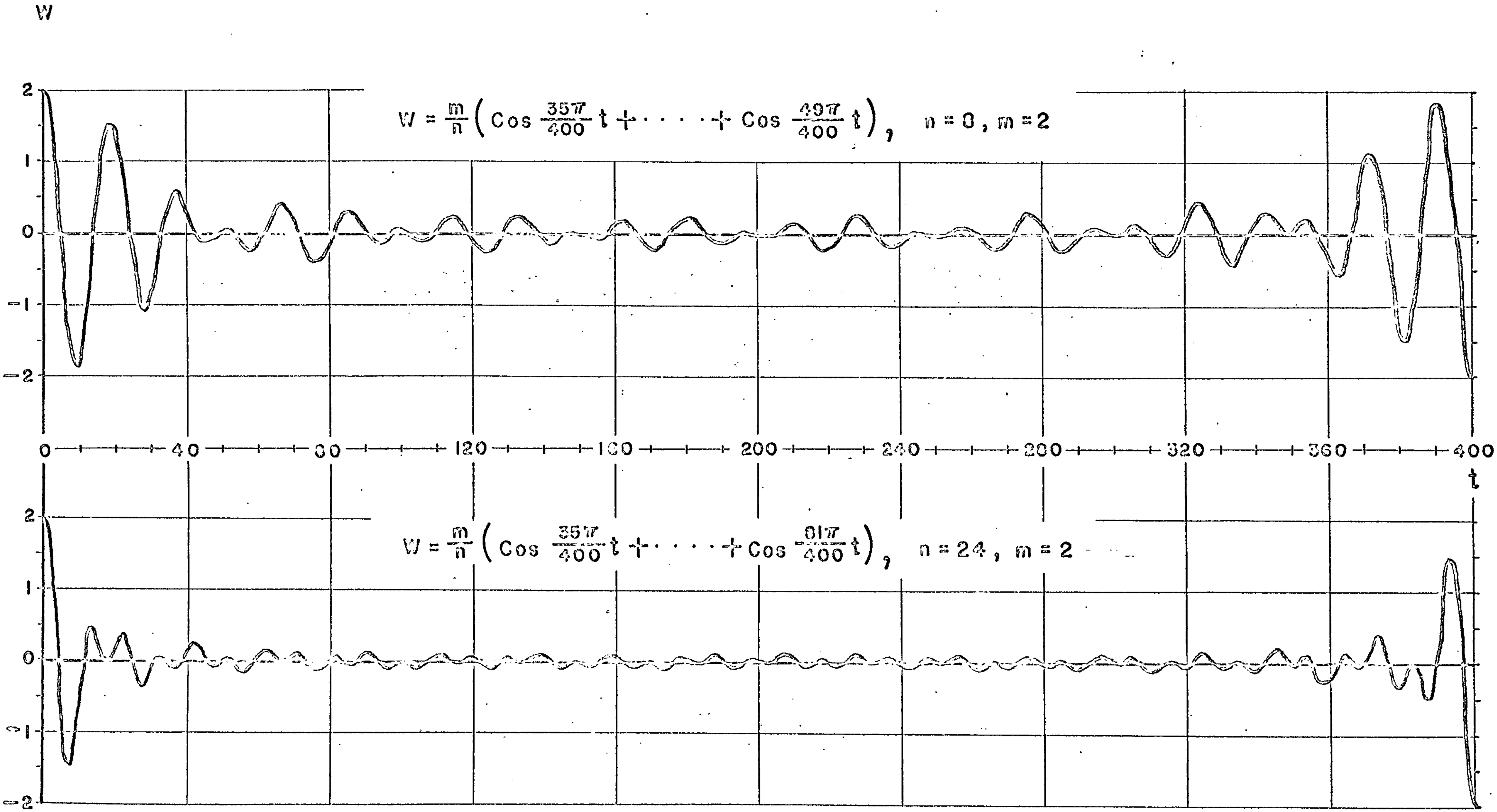

FIGURE 3b — EXAMPLES OF BEAT FUNCTIONS

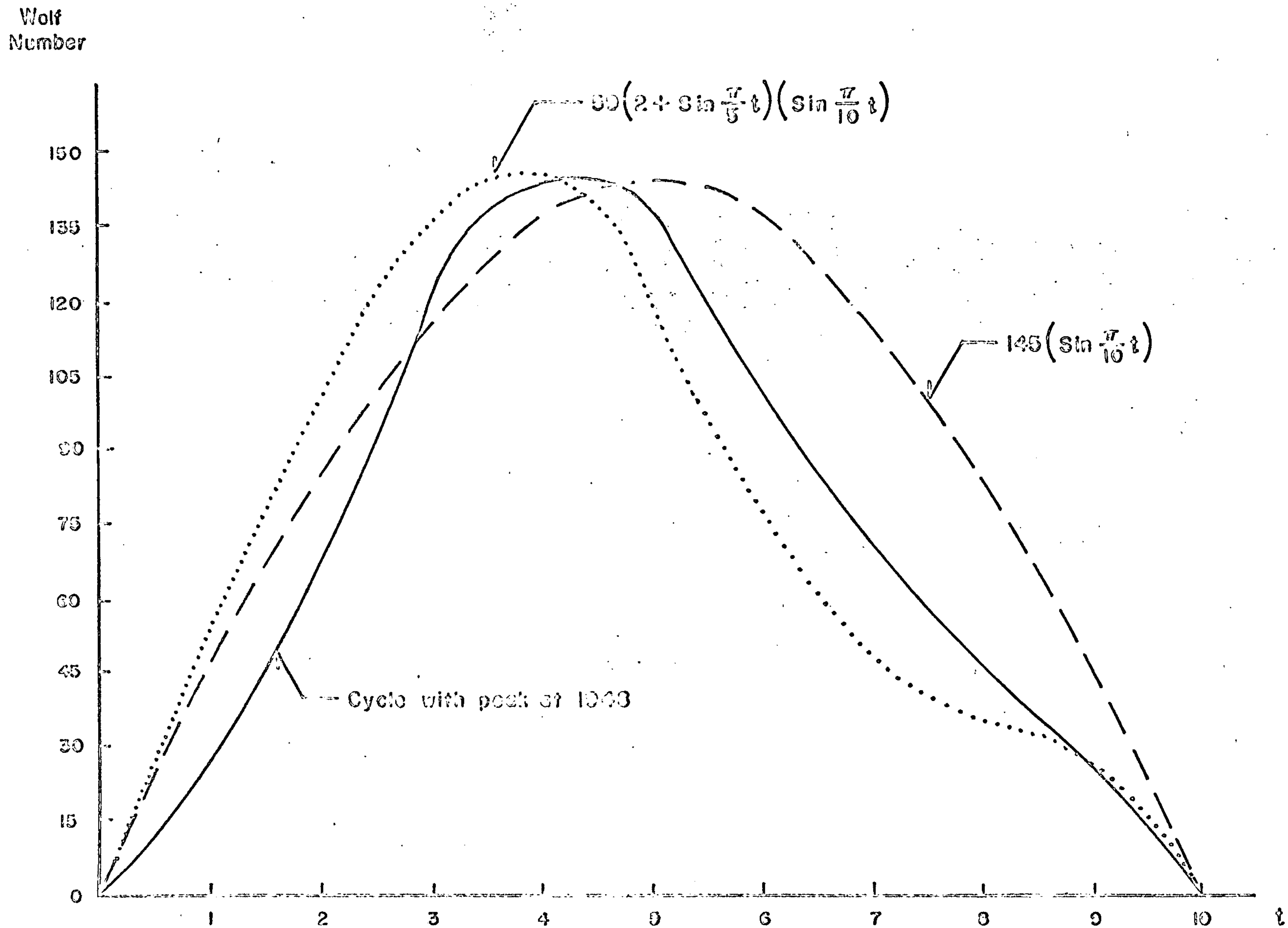

FIGURE 4a — EXAMPLE OF SKEWNESS ADJUSTMENT

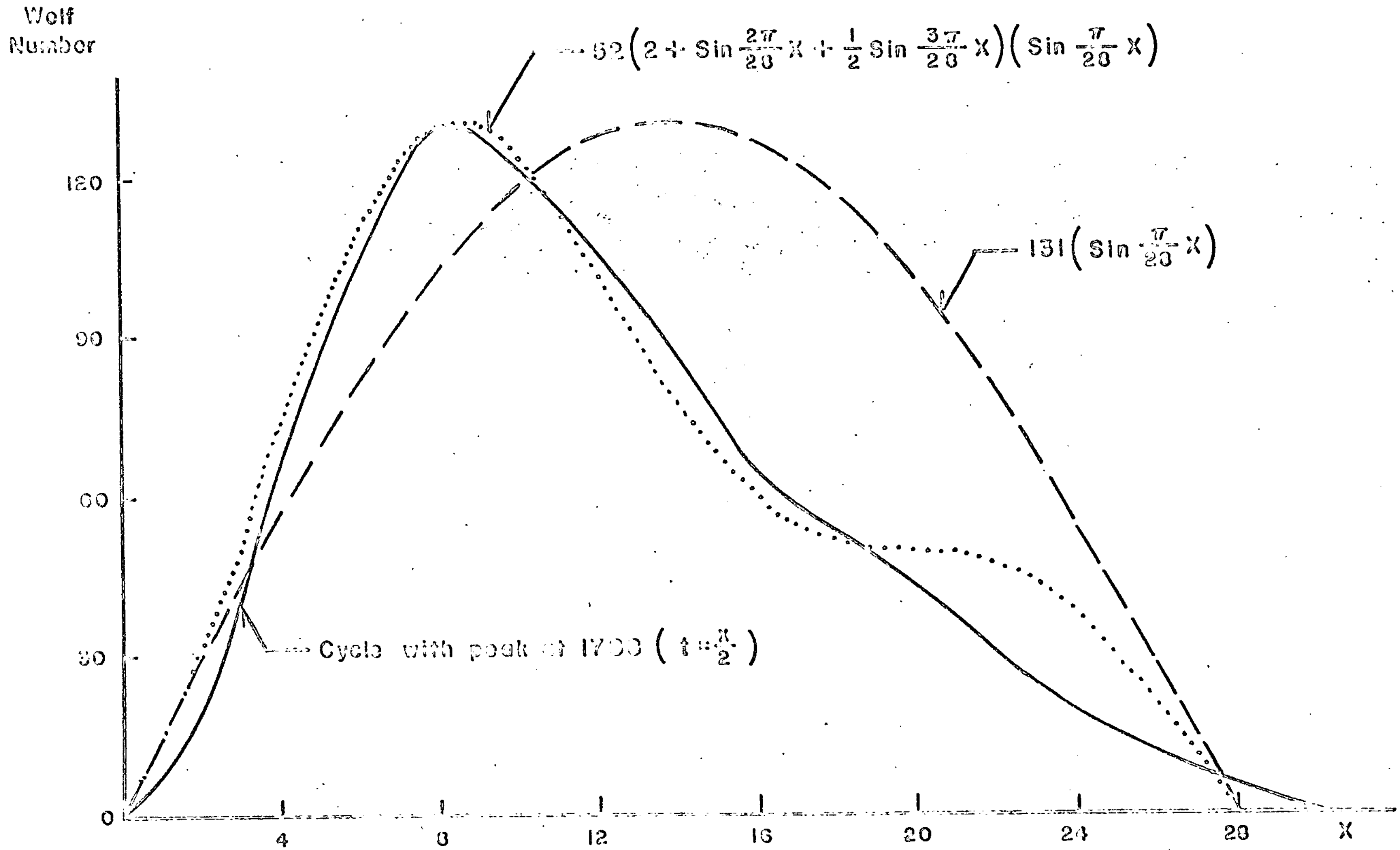

FIGURE 4b — EXAMPLE OF SKEWNESS ADJUSTMENT

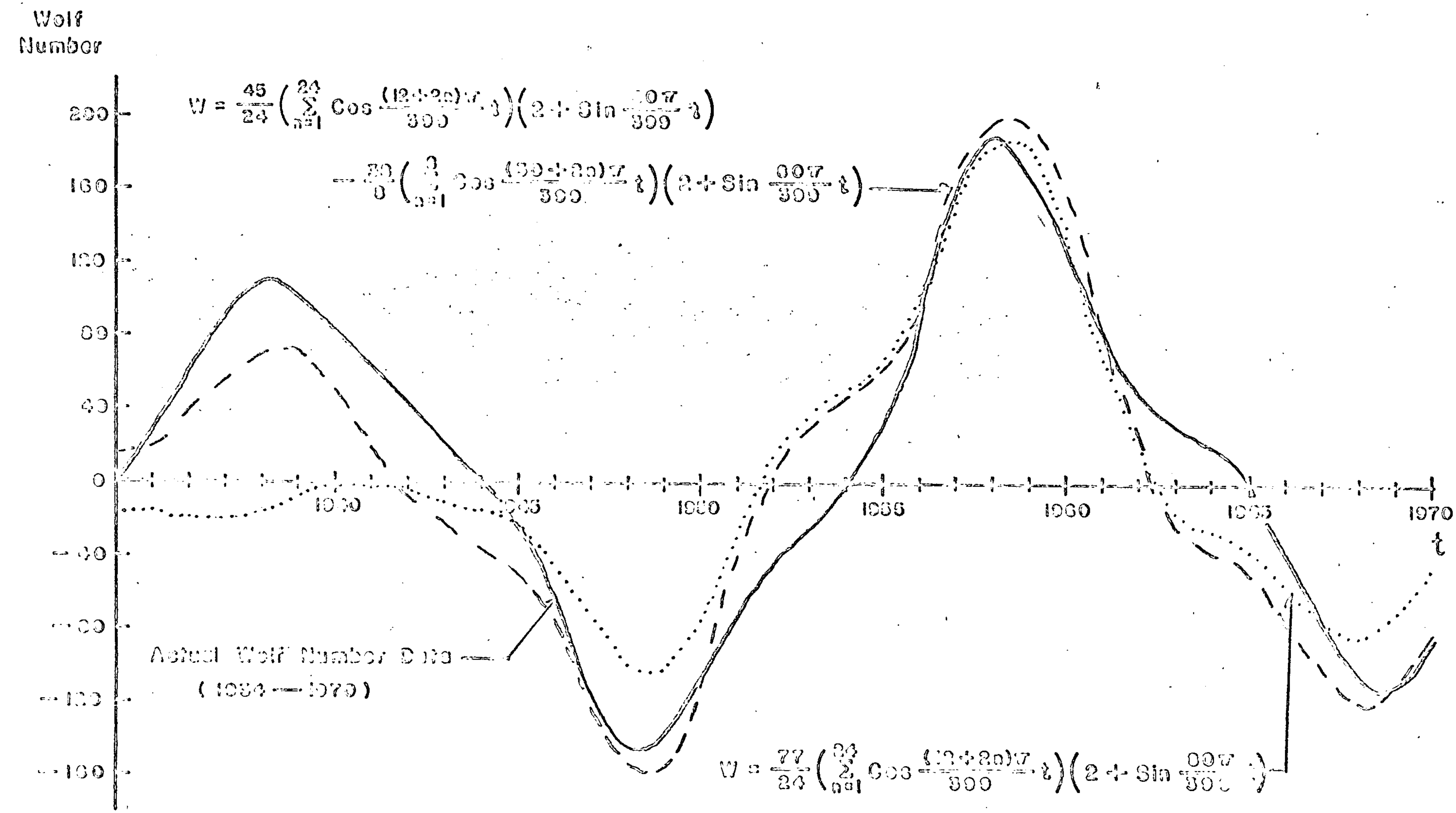

FIGURE 5 - EXAMPLE OF FIT OF ONE AND TWO FUNCTIONS

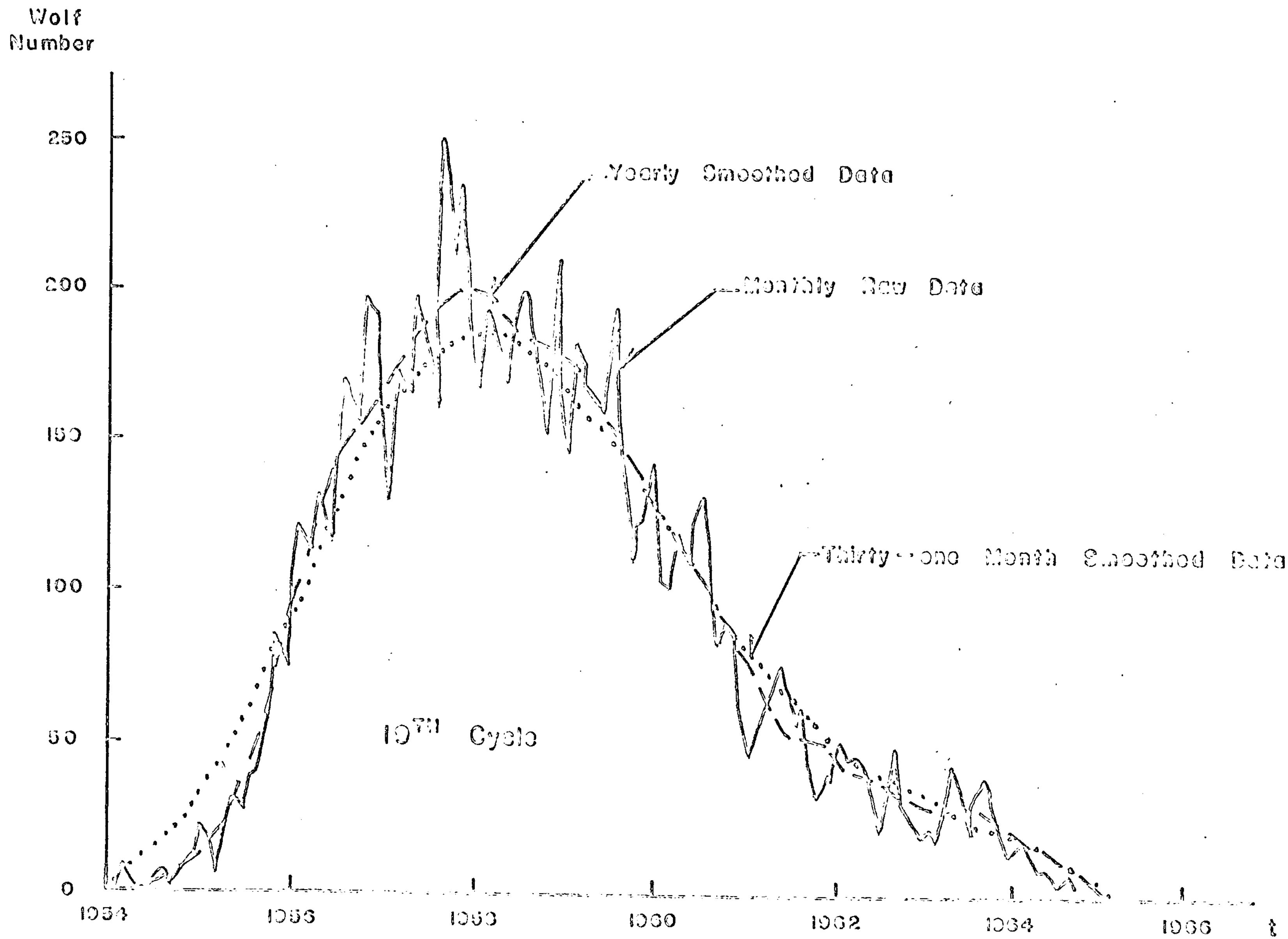

FIGURE 6 -- EXAMPLE OF DATA SMOOTHING

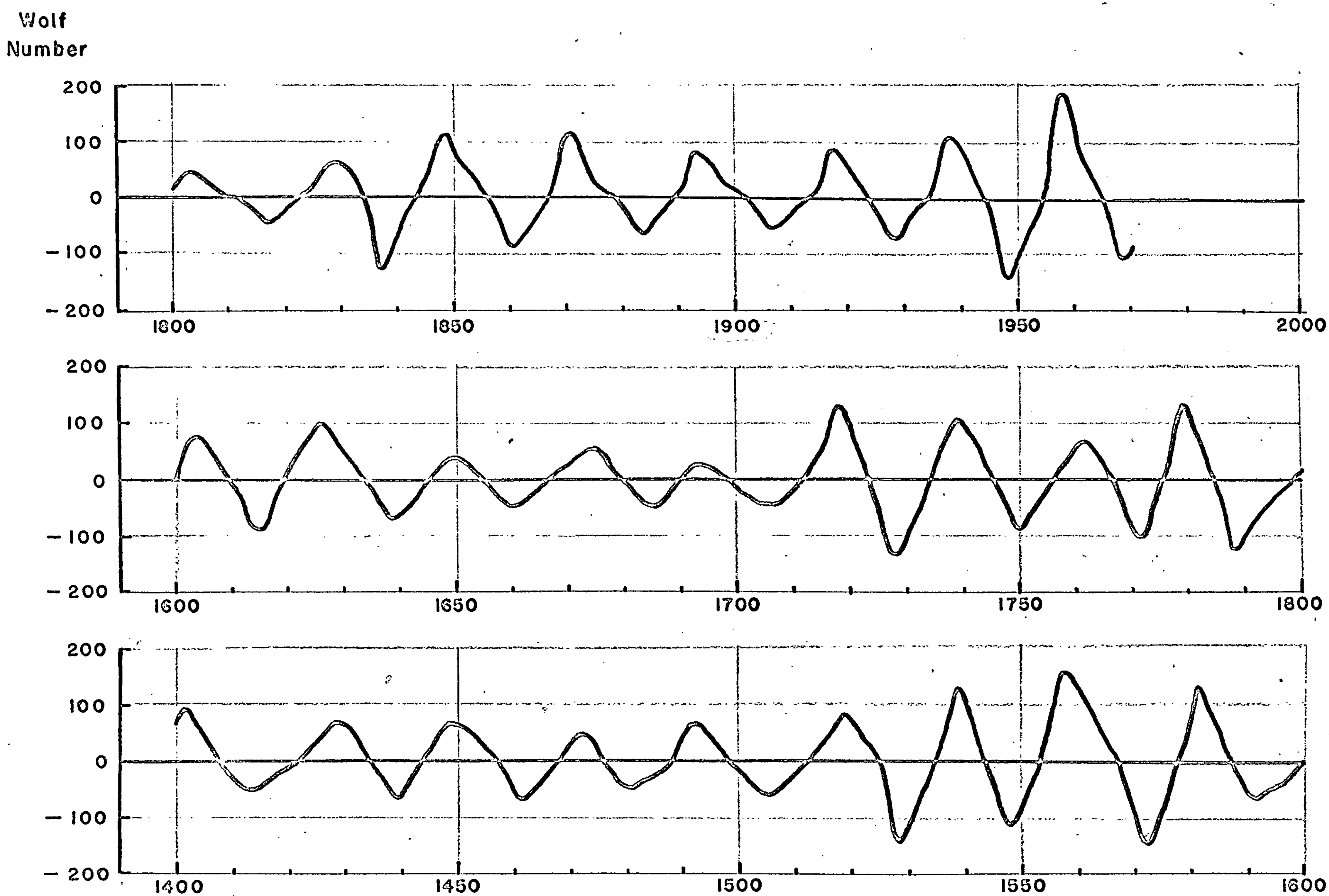

FIGURE 7 — WOLF NUMBER PLOT BASED ON SCHOVE DATA

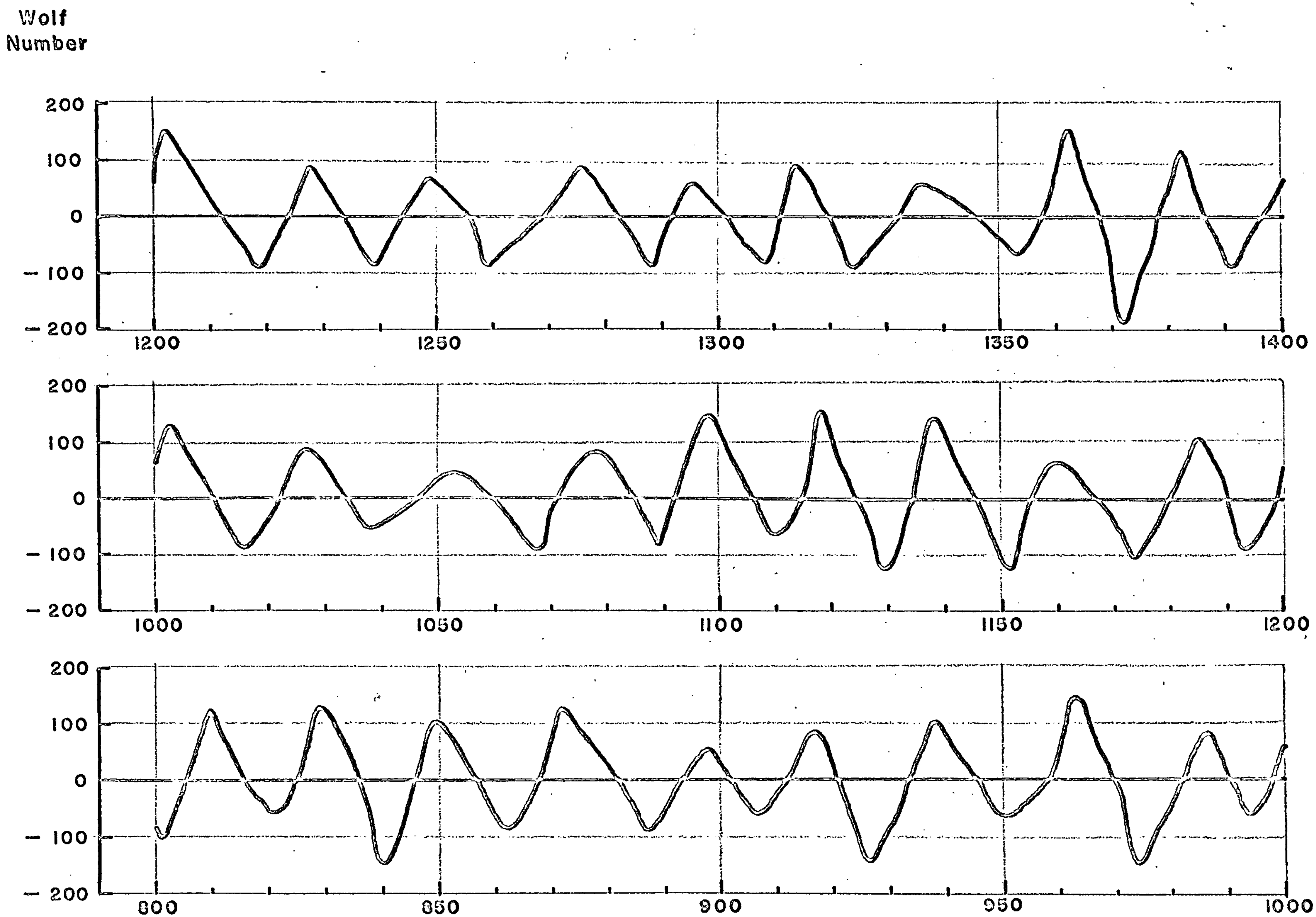

FIGURE 7 (CONT'D) — WOLF NUMBER PLOT BASED ON SCHOVE DATA

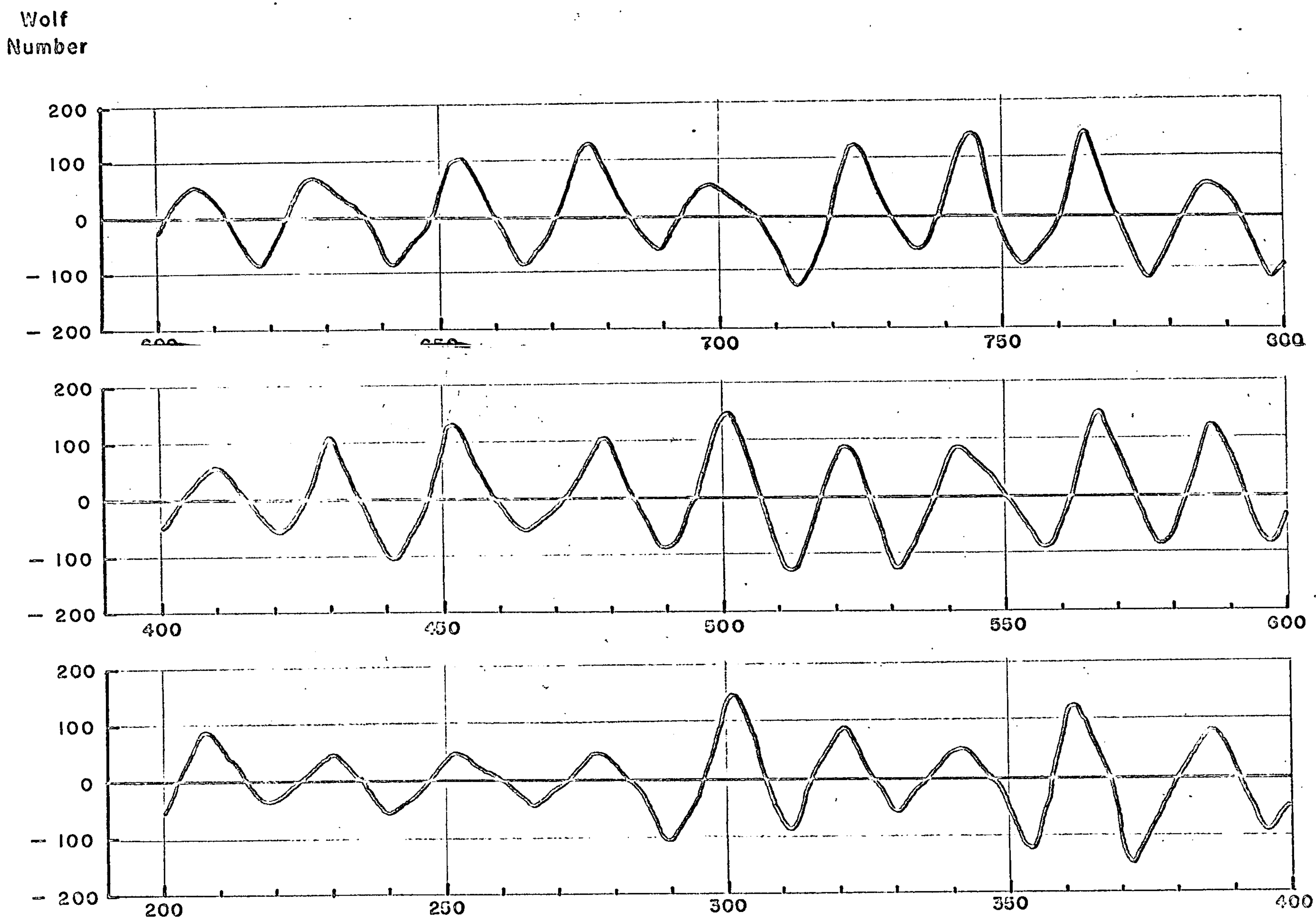

FIGURE 7(CONT'D) — WOLF NUMBER PLOT BASED ON SCHOVE DATA

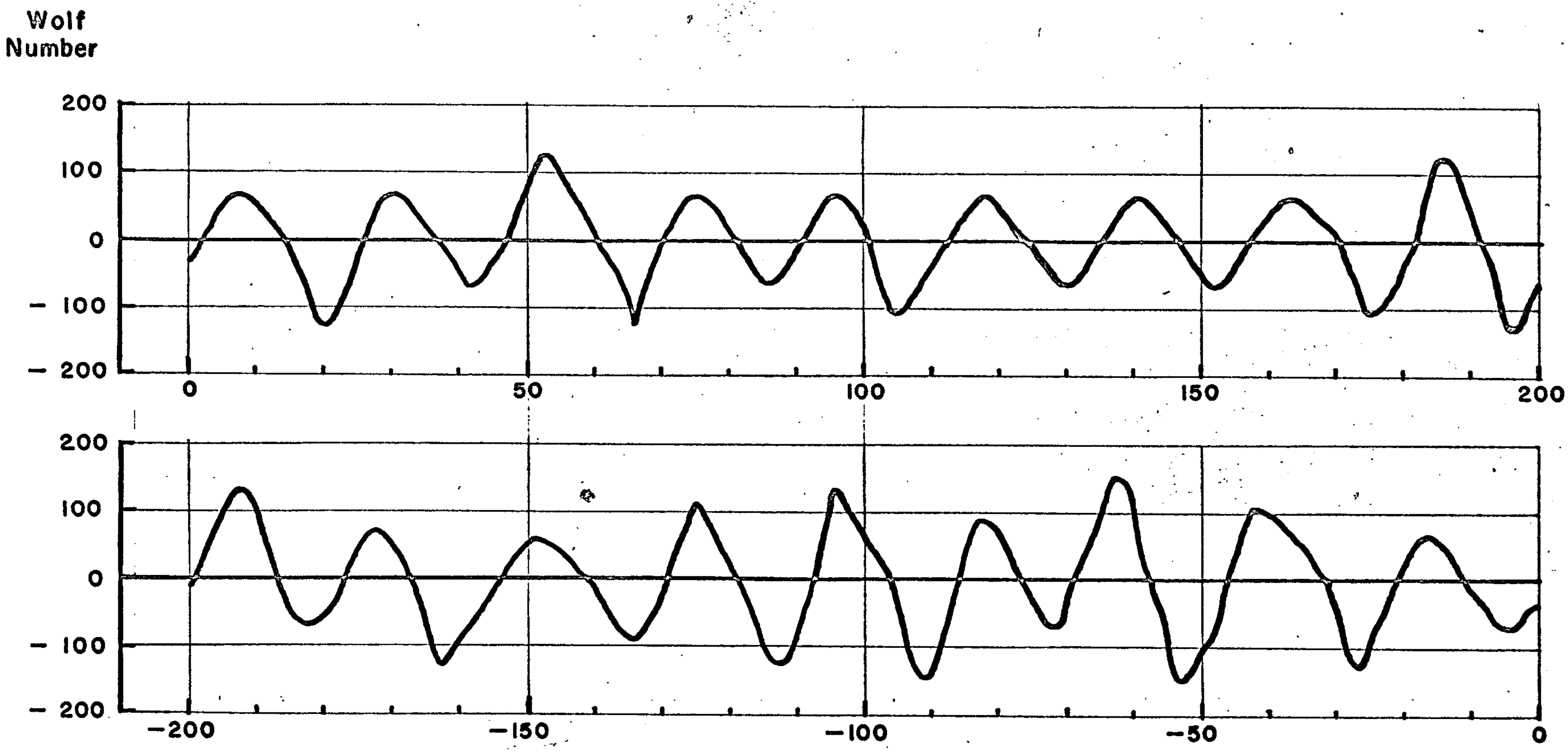

FIGURE 7 (CONT'D) — WOLF NUMBER PLOT BASED ON SCHOVE DATA

H 6 #

APPENDIX A

Values of Annual Wolf Numbers Used in Curve Fitting

Values for the modern period (1750 AD - 1970 AD) are derived from the thirty-one month smoothed data.

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
1970	- 85	1935	30	1900	10
1969	-110	1934	0	1899	20
1968	-105	1933	- 10	1898	30
1967	- 75	1932	- 20	1897	40
1966	- 35	1931	- 35	1896	55
1965	0	1930	- 55	1895	70
1964	20	1929	- 70	1894	80
1963	30	1928	- 75	1893	80
1962	50	1927	- 70	1892	60
1961	80	1926	- 60	1891	25
1960	130	1925	- 35	1890	10
1959	170	1924	- 10	1889	0
1958	190	1923	0	1888	- 10
1957	160	1922	20	1887	- 20
1956	95	1921	35	1886	- 35
1955	30	1920	55	1885	- 55
1954	0	1919	75	1884	- 60
1953	- 25	1918	85	1883	- 65
1952	- 45	1917	80	1882	- 55
1951	- 75	1916	60	1881	- 45
1950	-105	1915	30	1880	- 20
1949	-140	1914	10	1879	0
1948	-145	1913	0	1878	5
1947	-120	1912	- 10	1877	10
1946	- 65	1911	- 15	1876	15
1945	- 25	1910	- 30	1875	30
1944	0	1909	- 45	1874	55
1943	20	1908	- 55	1873	85
1942	40	1907	- 60	1872	110
1941	60	1906	- 60	1871	120
1940	80	1905	- 50	1870	105
1939	100	1904	- 35	1869	60
1938	110	1903	- 15	1868	25
1937	95	1902	- 5	1867	0
1936	60	1901	0	1866	- 20

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
1865	- 40	1820	- 20	1775	0
1864	- 50	1819	- 30	1774	- 30
1863	- 60	1818	- 35	1773	- 60
1862	- 70	1817	- 45	1772	-105
1861	- 90	1816	- 40	1771	-100
1860	- 90	1815	- 25	1770	- 95
1859	- 75	1814	- 15	1769	- 85
1858	- 45	1813	- 10	1768	- 50
1857	- 15	1812	- 5	1767	- 20
1856	0	1811	0	1766	0
1855	15	1810	5	1765	25
1854	30	1809	10	1764	40
1853	45	1808	15	1763	60
1852	60	1807	25	1762	70
1851	65	1806	40	1761	70
1850	85	1805	45	1760	60
1849	115	1804	50	1759	50
1848	110	1803	45	1758	45
1847	80	1802	40	1757	25
1846	55	1801	30	1756	0
1845	30	1800	15	1755	- 15
1844	15	1799	0	1754	- 25
1843	0	1798	- 5	1753	- 45
1842	- 20	1797	- 10	1752	- 60
1841	- 50	1796	- 20	1751	- 75
1840	- 70	1795	- 30	1750	- 90
1839	- 95	1794	- 45	1749	- 75
1838	-120	1793	- 55	1748	- 60
1837	-130	1792	- 65	1747	- 35
1836	-100	1791	- 80	1746	- 20
1835	- 40	1790	-105	1745	0
1834	0	1789	-125	1744	20
1833	20	1788	-130	1743	40
1832	40	1787	-100	1742	60
1831	60	1786	- 65	1741	75
1830	65	1785	- 20	1740	100
1829	65	1784	0	1739	110
1828	60	1783	30	1738	100
1827	40	1782	55	1737	75
1826	25	1781	80	1736	50
1825	15	1780	105	1735	25
1824	10	1779	135	1734	0
1823	0	1778	115	1733	- 30
1822	- 5	1777	65	1732	- 50
1821	- 10	1776	15	1731	- 80

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
1730	-100	1685	- 50	1640	- 65
1729	-130	1684	- 45	1639	- 70
1728	-140	1683	- 35	1638	- 65
1727	-130	1682	- 30	1637	- 50
1726	-100	1681	- 20	1636	- 30
1725	- 75	1680	- 10	1635	- 15
1724	- 35	1679	0	1634	0
1723	0	1678	15	1633	15
1722	30	1677	35	1632	30
1721	55	1676	55	1631	45
1720	75	1675	60	1630	55
1719	120	1674	55	1629	70
1718	130	1673	50	1628	80
1717	120	1672	45	1627	95
1716	95	1671	35	1626	100
1715	70	1670	30	1625	95
1714	50	1669	20	1624	80
1713	25	1668	15	1623	60
1712	0	1667	5	1622	45
1711	- 5	1666	0	1621	30
1710	- 15	1665	- 10	1620	15
1709	- 25	1664	- 20	1619	0
1708	- 30	1663	- 30	1618	- 30
1707	- 40	1662	- 35	1617	- 60
1706	- 45	1661	- 45	1616	- 85
1705	- 50	1660	- 50	1615	- 90
1704	- 45	1659	- 45	1614	- 85
1703	- 40	1658	- 35	1613	- 60
1702	- 30	1657	- 20	1612	- 40
1701	- 25	1656	- 10	1611	- 20
1700	- 15	1655	0	1610	0
1699	- 10	1654	5	1609	20
1698	0	1653	15	1608	30
1697	5	1652	25	1607	45
1696	15	1651	30	1606	60
1695	20	1650	35	1605	75
1694	25	1649	40	1604	80
1693	30	1648	35	1603	75
1692	25	1647	25	1602	50
1691	15	1646	10	1601	25
1690	10	1645	0	1600	0
1689	0	1644	- 15	1599	- 10
1688	- 15	1643	- 30	1598	- 20
1687	- 30	1642	- 40	1597	- 25
1686	- 45	1641	- 55	1596	- 35

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
1595	-45	1550	-80	1505	-60
1594	-55	1549	-110	1504	-55
1593	-60	1548	-120	1503	-45
1592	-65	1547	-110	1502	-35
1591	-70	1546	-80	1501	-35
1590	-65	1545	-50	1500	-20
1589	-40	1544	-25	1499	-10
1588	-20	1543	0	1498	0
1587	0	1542	40	1497	15
1586	20	1541	75	1496	25
1585	50	1540	115	1495	40
1584	75	1539	130	1494	50
1583	95	1538	115	1493	65
1582	125	1537	75	1492	70
1581	130	1536	40	1491	65
1580	100	1535	0	1490	50
1579	50	1534	-20	1489	25
1578	0	1533	-50	1488	0
1577	-30	1532	-70	1487	-5
1576	-60	1531	-100	1486	-15
1575	-90	1530	-120	1485	-20
1574	-115	1529	-145	1484	-25
1573	-145	1528	-150	1483	-35
1572	-150	1527	-120	1482	-40
1571	-130	1526	-60	1481	-45
1570	-100	1525	0	1480	-50
1569	-60	1524	15	1479	-45
1568	-30	1523	25	1478	-30
1567	0	1522	40	1477	-15
1566	20	1521	55	1476	0
1565	40	1520	75	1475	15
1564	60	1519	80	1474	30
1563	80	1518	75	1473	45
1562	100	1517	60	1472	50
1561	120	1516	50	1471	45
1560	135	1515	35	1470	30
1559	155	1514	25	1469	15
1558	160	1513	10	1468	0
1557	155	1512	0	1467	-10
1556	110	1511	-10	1466	-20
1555	65	1510	-20	1465	-35
1554	30	1509	-30	1464	-45
1553	0	1508	-35	1463	-55
1552	-30	1507	-50	1462	-65
1551	-60	1506	-55	1461	-70

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
1460	- 55	1415	- 40	1370	-110
1459	- 35	1414	- 45	1369	- 50
1458	- 20	1413	- 50	1368	0
1457	0	1412	- 45	1367	30
1456	10	1411	- 35	1366	45
1455	20	1410	- 30	1365	85
1454	30	1409	- 20	1364	110
1453	40	1408	- 10	1363	145
1452	50	1407	0	1362	150
1451	55	1406	20	1361	130
1450	65	1405	40	1360	85
1449	70	1404	60	1359	40
1448	65	1403	80	1358	0
1447	55	1402	90	1357	- 15
1446	40	1401	80	1356	- 30
1445	25	1400	70	1355	- 45
1444	15	1399	50	1354	- 60
1443	0	1398	30	1353	- 70
1442	- 20	1397	15	1352	- 65
1441	- 40	1396	0	1351	- 50
1440	- 60	1395	- 20	1350	- 40
1439	- 70	1394	- 40	1349	- 30
1438	- 60	1393	- 60	1348	- 20
1437	- 45	1392	- 80	1347	- 10
1436	- 30	1391	- 90	1346	0
1435	- 15	1390	- 80	1345	5
1434	0	1389	- 60	1344	15
1433	20	1388	- 40	1343	20
1432	35	1387	- 20	1342	25
1431	50	1386	0	1341	35
1430	65	1385	35	1340	40
1429	70	1384	50	1339	45
1428	65	1383	105	1338	50
1427	55	1382	110	1337	55
1426	45	1381	105	1336	60
1425	35	1380	70	1335	55
1424	30	1379	35	1334	40
1423	15	1378	0	1333	20
1422	10	1377	- 35	1332	0
1421	0	1376	- 70	1331	- 15
1420	- 5	1375	-110	1330	- 25
1419	- 15	1374	-140	1329	- 40
1418	- 20	1373	-180	1328	- 50
1417	- 25	1372	-190	1327	- 60
1416	- 35	1371	-180	1326	- 70

Year	Wolf Number
1325	- 85
1324	- 90
1323	- 80
1322	- 60
1321	- 40
1320	- 20
1319	0
1318	20
1317	45
1316	60
1315	85
1314	90
1313	85
1312	45
1311	0
1310	- 35
1309	- 80
1308	- 90
1307	- 80
1306	- 70
1305	- 55
1304	- 40
1303	- 25
1302	- 15
1301	0
1300	15
1299	30
1298	45
1297	55
1296	60
1295	55
1294	45
1293	30
1292	15
1291	0
1290	- 40
1289	- 85
1288	- 90
1287	- 85
1286	- 65
1285	- 45
1284	- 30
1283	- 15
1282	0
1281	15
1280	35
1279	50
1278	65
1277	85
1276	90
1275	85
1274	70
1273	55
1272	40
1271	25
1270	15
1269	0
1268	- 10
1267	- 20
1266	- 30
1265	- 40
1264	- 50
1263	- 60
1262	- 65
1261	- 75
1260	- 85
1259	- 90
1258	- 80
1257	- 40
1256	0
1255	10
1254	20
1253	35
1252	45
1251	55
1250	65
1249	70
1248	65
1247	45
1246	30
1245	15
1244	0
1243	- 25
1242	- 40
1241	- 60
1240	- 80
1239	- 90
1238	- 80
1237	- 60
1236	- 45
1235	- 30
1234	- 15
1233	0
1232	20
1231	45
1230	60
1229	80
1228	90
1227	80
1226	55
1225	25
1224	0
1223	- 20
1222	- 40
1221	- 60
1220	- 80
1219	- 90
1218	- 85
1217	- 70
1216	- 55
1215	- 40
1214	- 30
1213	- 15
1212	0
1211	15
1210	30
1209	50
1208	65
1207	80
1206	95
1205	110
1204	125
1203	145
1202	150
1201	130
1200	60
1199	0
1198	- 20
1197	- 35
1196	- 50
1195	- 70
1194	- 85
1193	- 90
1192	- 80
1191	- 30

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
1190	0	1145	0	1100	120
1189	30	1144	25	1099	145
1188	60	1143	50	1098	150
1187	85	1142	70	1097	135
1186	105	1141	90	1096	110
1185	110	1140	115	1095	80
1184	105	1139	145	1094	55
1183	80	1138	150	1093	30
1182	55	1137	130	1092	0
1181	30	1136	90	1091	-30
1180	0	1135	35	1090	-55
1179	-15	1134	0	1089	-85
1178	-35	1133	-40	1088	-90
1177	-55	1132	-65	1087	-80
1176	-70	1131	-100	1086	-65
1175	-85	1130	-125	1085	-50
1174	-105	1129	-130	1084	-30
1173	-110	1128	-110	1083	-15
1172	-100	1127	-80	1082	0
1171	-75	1126	-55	1081	30
1170	-55	1125	-25	1080	55
1169	-35	1124	0	1079	85
1168	-20	1123	25	1078	90
1167	0	1122	50	1077	85
1166	10	1121	75	1076	75
1165	25	1120	110	1075	60
1164	35	1119	145	1074	45
1163	45	1118	150	1073	30
1162	55	1117	145	1072	15
1161	65	1116	70	1071	0
1160	70	1115	0	1070	-25
1159	65	1114	-20	1069	-60
1158	50	1113	-40	1068	-85
1157	30	1112	-55	1067	-90
1156	15	1111	-65	1066	-85
1155	0	1110	-70	1065	-75
1154	-45	1109	-65	1064	-60
1153	-90	1108	-45	1063	-45
1152	-125	1107	-20	1062	-30
1151	-130	1106	0	1061	-15
1150	-120	1105	20	1060	0
1149	-90	1104	40	1059	5
1148	-70	1103	60	1058	15
1147	-45	1102	80	1057	20
1146	-20	1101	100	1056	25

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
1055	35	1010	0	965	125
1054	40	1009	20	964	145
1053	45	1008	40	963	150
1052	50	1007	60	962	145
1051	45	1006	80	961	100
1050	35	1005	105	960	55
1049	25	1004	120	959	0
1048	15	1003	130	958	- 5
1047	0	1002	125	957	- 15
1046	- 5	1001	95	956	- 25
1045	- 15	1000	65	955	- 35
1044	- 25	999	30	954	- 40
1043	- 30	998	0	953	- 50
1042	- 35	997	- 20	952	- 60
1041	- 45	996	- 35	951	- 65
1040	- 50	995	- 55	950	- 70
1039	- 55	994	- 60	949	- 65
1038	- 60	993	- 55	948	- 50
1037	- 50	992	- 35	947	- 35
1036	- 35	991	- 15	946	- 20
1035	- 15	990	0	945	0
1034	0	989	35	944	15
1033	15	988	55	943	35
1032	25	987	85	942	50
1031	40	986	90	941	65
1030	55	985	85	940	85
1029	70	984	60	939	105
1028	80	983	30	938	110
1027	90	982	0	937	105
1026	85	981	- 20	936	80
1025	70	980	- 40	935	40
1024	45	979	- 60	934	0
1023	20	978	- 80	933	- 20
1022	0	977	-105	932	- 40
1021	- 15	976	-125	931	- 60
1020	- 35	975	-145	930	- 80
1019	- 50	974	-150	929	-100
1018	- 70	973	-140	928	-120
1017	- 85	942	- 95	927	-140
1016	- 90	971	- 50	926	-150
1015	- 85	970	0	925	-140
1014	- 70	969	0 25	924	-100
1013	- 50	968	45	923	- 70
1012	- 35	967	70	922	- 40
1011	- 20	966	95	921	0

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
920	25	875	95	830	125
919	55	874	110	829	130
918	80	873	125	828	120
917	90	872	130	827	80
916	85	871	105	826	40
915	70	870	70	825	0
914	45	869	35	824	- 15
913	25	868	0	823	- 35
912	0	867	- 20	822	- 55
911	- 15	866	- 35	821	- 60
910	- 30	865	- 50	820	- 55
909	- 40	864	- 65	819	- 40
908	- 55	863	- 85	818	- 35
907	- 60	862	- 90	817	- 25
906	- 55	861	- 85	816	- 15
905	- 45	860	- 70	815	0
904	- 30	859	- 50	814	30
903	- 15	858	- 35	813	55
902	0	857	- 15	812	75
901	15	856	0	811	100
900	35	855	20	810	125
899	55	854	40	809	130
898	60	853	60	808	125
897	55	852	80	807	95
896	45	851	100	806	60
895	30	850	110	805	30
894	15	849	105	804	0
893	0	848	70	803	- 20
892	- 15	847	30	802	- 45
891	- 35	846	0	801	- 65
890	- 50	845	- 25	800	- 85
889	- 65	844	- 55	799	-105
888	- 85	843	- 85	798	-110
887	- 90	842	-110	797	-100
886	- 85	841	-145	796	- 80
885	- 65	840	-150	795	- 55
884	- 45	839	-140	794	- 30
883	- 20	838	- 90	793	0
882	0	837	- 40	792	10
881	15	836	0	791	20
880	25	835	25	790	30
879	40	834	50	789	45
878	55	833	70	788	55
877	70	832	90	787	60
876	80	831	110	786	55

Year	Wolf Number
785	45
784	30
783	15
782	0
781	- 20
780	- 40
779	- 65
778	- 85
777	-105
776	-110
775	-105
774	- 90
773	- 60
772	- 40
771	- 20
770	0
769	40
768	70
767	110
766	145
765	150
764	145
763	95
762	50
761	0
760	- 15
759	- 30
758	- 45
757	- 60
756	- 70
755	- 85
754	- 90
753	- 85
752	- 70
751	- 45
750	- 20
749	0
748	40
747	100
746	145
745	150
744	145
743	120
742	90
741	60

Year	Wolf Number
740	35
739	0
738	- 15
737	- 35
736	- 55
735	- 60
734	- 55
733	- 40
732	- 25
731	- 15
730	0
729	25
728	50
727	80
726	100
725	125
724	130
723	125
722	100
721	65
720	35
719	0
718	- 30
717	- 65
716	-100
715	-125
714	-130
713	-125
712	-105
711	- 80
710	- 60
709	- 40
708	- 20
707	0
706	5
705	15
704	20
703	30
702	40
701	45
700	55
699	60
698	55
697	45
696	30

Year	Wolf Number
695	20
694	10
693	0
692	- 20
691	- 40
690	- 55
689	- 60
688	- 55
687	- 40
686	- 30
685	- 15
684	0
683	20
682	40
681	60
680	80
679	105
678	125
677	130
676	125
675	100
674	75
673	50
672	25
671	0
670	- 20
669	- 35
668	- 50
667	- 70
666	- 85
665	- 90
664	- 85
663	- 65
662	- 45
661	- 25
660	0
659	20
658	40
657	65
656	85
655	105
654	110
653	105
652	90
651	60

Year	Wolf Number
650	30
649	0
648	- 15
647	- 30
646	- 40
645	- 60
644	- 70
643	- 85
642	- 90
641	- 85
640	- 70
639	- 50
638	- 30
637	0
636	15
635	20
634	25
633	35
632	45
631	55
630	65
629	75
628	80
627	75
626	60
625	40
624	20
623	0
622	- 20
621	- 45
620	- 65
619	- 85
618	- 90
617	- 85
616	- 60
615	- 40
614	- 20
613	0
612	10
611	20
610	35
609	45
608	55
607	60
606	55

Year	Wolf Number
605	40
604	30
603	15
602	0
601	- 15
600	- 30
599	- 55
598	- 75
597	- 80
596	- 75
595	- 55
594	- 40
593	- 20
592	0
591	30
590	60
589	95
588	125
587	130
586	125
585	90
584	60
583	35
582	0
581	- 25
580	- 55
579	- 85
578	- 90
577	- 85
576	- 65
575	- 45
574	- 20
573	0
572	30
541	60
570	90
569	120
568	145
567	150
566	145
565	110
564	70
563	35
562	0
561	- 25

Year	Wolf Number
560	- 45
559	- 70
558	- 85
557	- 90
556	- 85
555	- 70
554	- 55
553	- 35
552	- 20
551	0
550	10
549	20
548	30
547	40
546	55
545	65
544	75
543	85
542	90
541	85
540	55
539	30
538	0
537	- 20
536	- 40
535	- 65
534	- 85
533	-105
532	-125
531	-130
530	-125
529	- 95
528	- 60
527	- 30
526	0
525	30
524	50
523	85
522	90
521	85
520	60
519	40
518	20
517	0
516	- 30

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
515	- 65	470	- 15	425	- 15
514	-100	469	- 25	424	- 30
513	-125	468	- 35	423	- 40
512	-130	467	- 45	422	- 55
511	-125	466	- 55	421	- 60
510	-100	465	- 60	420	- 55
509	- 60	464	- 55	419	- 40
508	- 30	463	- 45	418	- 30
507	0	462	- 35	417	- 15
506	30	461	- 25	416	0
505	60	460	- 15	415	10
504	90	459	0	414	20
503	120	458	20	413	35
502	145	457	40	412	45
501	150	456	60	411	55
500	145	455	80	410	60
499	120	454	100	409	55
498	90	453	125	408	45
497	60	452	130	407	35
496	30	451	125	406	25
495	0	450	80	405	10
494	- 20	449	40	404	0
493	- 40	448	0	403	- 15
492	- 60	447	- 15	402	- 25
491	- 85	446	- 30	401	- 35
490	- 90	445	- 50	400	- 50
489	- 85	444	- 70	399	- 60
488	- 70	443	- 90	398	- 75
487	- 50	442	-105	397	- 85
486	- 35	441	-110	396	- 90
485	- 15	440	-105	395	- 85
484	0	439	- 80	394	- 65
483	25	438	- 65	393	- 45
482	50	437	- 40	392	- 25
481	75	436	- 20	391	0
480	105	435	0	390	20
479	110	434	25	389	50
478	105	433	50	388	85
477	90	432	80	387	90
476	70	431	105	386	85
475	55	430	110	385	75
474	40	429	105	384	60
473	20	428	65	383	45
472	0	427	30	382	30
471	- 10	426	0	381	15

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
380	0	335	- 10	290	-110
379	- 20	334	- 20	289	-105
378	- 40	333	- 35	288	- 80
377	- 65	332	- 45	287	- 60
376	- 80	331	- 55	286	- 45
375	-105	330	- 60	285	- 25
374	-125	329	- 50	284	0
373	-145	328	- 40	283	5
372	-150	327	- 20	282	10
371	-145	326	0	281	30
370	-100	325	20	280	35
369	- 45	324	40	279	40
368	0	323	60	278	45
367	25	322	85	277	50
366	50	321	90	276	45
365	75	320	85	275	35
364	100	319	70	274	20
363	125	318	55	273	10
362	130	317	35	272	0
361	125	316	20	271	- 5
360	80	315	0	270	- 10
359	40	314	- 30	269	- 15
358	0	313	- 60	268	- 25
357	- 40	312	- 85	267	- 30
356	- 80	311	- 90	266	- 35
355	-125	310	- 85	265	- 40
354	-130	309	- 55	264	- 35
353	-125	308	- 30	263	- 20
352	-100	307	0	262	- 10
351	- 75	306	35	261	- 5
350	- 50	305	70	260	0
349	- 25	304	110	259	5
348	0	303	145	258	10
347	10	302	150	257	15
346	20	301	145	256	20
345	35	300	115	255	30
344	45	299	85	254	35
343	55	298	60	253	45
342	60	297	30	252	50
341	55	296	0	251	45
340	45	295	- 20	250	35
339	35	294	- 40	249	20
338	25	293	- 65	248	10
337	15	292	- 85	247	0
336	0	291	-105	246	- 10

Year	Wolf Number	Year	Wolf Number	Year	Wolf Number
245	- 20	200	- 60	155	- 35
244	- 30	199	- 80	154	- 50
243	- 40	198	-100	153	- 65
242	- 50	197	-125	152	- 70
241	- 55	196	-130	151	- 65
240	- 60	195	-125	150	- 55
239	- 55	194	- 85	149	- 40
238	- 40	193	- 50	148	- 30
237	- 30	192	0	147	- 15
236	- 15	191	25	146	0
235	0	190	45	145	15
234	10	189	75	144	30
233	25	188	100	143	50
232	30	187	125	142	65
231	45	186	130	141	70
230	50	185	125	140	65
229	45	184	85	139	55
228	35	183	45	138	40
227	20	182	0	137	30
226	10	181	- 15	136	15
225	0	180	- 35	135	0
224	- 5	179	- 50	134	- 15
223	- 10	178	- 70	133	- 30
222	- 15	177	- 85	132	- 50
221	- 25	176	-105	131	- 65
220	- 35	175	-110	130	- 70
219	- 40	174	-105	129	- 65
218	- 35	173	- 80	128	- 55
217	- 25	172	- 55	127	- 40
216	- 15	171	- 30	126	- 30
215	- 5	170	0	125	- 15
214	0	169	10	124	0
213	15	168	20	123	10
212	35	167	35	122	25
211	50	166	45	121	40
210	70	165	55	120	55
209	85	164	65	119	65
208	90	163	70	118	70
207	85	162	65	117	65
206	70	161	55	116	55
205	45	160	40	115	40
204	25	159	25	114	25
203	0	158	15	113	15
202	- 15	157	0	112	0
201	- 35	156	- 15	111	- 15

Year	Wolf Number	Year	Wolf Number		Year	Wolf Number
110	- 35	65	- 85		20	-130
109	- 55	64	- 60		19	-125
108	- 70	63	- 45		18	- 90
107	- 90	62	- 30		17	- 60
106	-105	61	- 5		16	- 35
105	-110	60	0		15	0
104	-105	59	20		14	10
103	- 75	58	40		13	20
102	- 40	57	65		12	30
101	0	56	85		11	40
100	15	55	105		10	55
99	35	54	125		09	65
98	50	53	130		08	70
97	65	52	125		07	65
96	70	51	105		06	50
95	65	50	80		05	35
94	55	49	55		04	15
93	35	48	30		03	0
92	20	47	0		02	- 10
91	0	46	- 15		01	- 20
90	- 15	45	- 35		0	- 40
89	- 30	44	- 50	B.C.	-1	- 45
88	- 50	43	- 65		-2	- 50
87	- 65	42	- 70		-3	- 60
86	- 70	41	- 65		-4	- 70
85	- 65	40	- 50		-5	- 65
84	- 55	39	- 35		-6	- 60
83	- 40	38	- 20		-7	- 50
82	- 25	37	0		-8	- 45
81	- 10	36	10		-9	- 35
80	0	35	25		-10	- 20
79	25	34	40		-11	0
78	40	33	50		-12	25
77	65	32	65		-13	40
76	70	31	70		-14	55
75	65	30	60		-15	65
74	55	29	55		-16	70
73	40	28	35		-17	65
72	25	27	15		-18	60
71	10	26	0		-19	45
70	0	25	- 25		-20	25
69	- 35	24	- 50		-21	0
68	- 70	23	- 75		-22	- 25
67	-100	22	-100		-23	- 50
66	-125	21	-125		-24	- 75

Year	Wolf Number
-25	-100
-26	-125
-27	-130
-28	-120
-29	- 90
-30	- 60
-31	- 30
-32	0
-33	20
-34	30
-35	45
-36	60
-37	70
-38	80
-39	90
-40	100
-41	105
-42	110
-43	95
-44	80
-45	50
-46	0
-47	- 50
-48	- 80
-49	- 95
-50	-115
-51	-130
-52	-140
-53	-150
-54	-130
-55	- 95
-56	- 60
-57	- 30
-58	0
-59	50
-60	110
-61	140
-62	150
-63	140
-64	120
-65	95
-66	75
-67	50
-68	25
-69	0
-70	- 40
-71	- 65
-72	- 70
-73	- 65
-74	- 50
-75	- 35
-76	- 20
-77	0
-78	20
-79	55
-80	70
-81	80
-82	90
-83	80
-84	60
-85	25
-86	0
-87	- 40
-88	- 75
-89	-115
-90	-140
-91	-150
-92	-140
-93	-100
-94	- 60
-95	- 20
-96	0
-97	20
-98	35
-99	55
-100	60
-101	95
-102	110
-103	125
-104	130
-105	110
-106	70
-107	30
-108	0
-109	- 40
-110	- 80
-111	-110
-112	-125
-113	-130
-114	-125
-115	-110
-116	- 85
-117	- 50
-118	- 25
-119	0
-120	20
-121	40
-122	60
-123	80
-124	100
-125	110
-126	100
-127	70
-128	40
-129	0
-130	- 20
-131	- 50
-132	- 70
-133	- 85
-134	- 90
-135	- 90
-136	- 85
-137	- 75
-138	- 65
-139	- 45
-140	- 20
-141	0
-142	10
-143	20
-144	30
-145	40
-146	45
-147	50
-148	55
-149	60
-150	55
-151	50
-152	35
-153	15
-154	0
-155	- 15
-156	- 30
-157	- 45
-158	- 60
-159	- 75

Year	Wolf Number
-160	- 95
-161	-110
-162	-125
-163	-130
-164	-110
-165	- 80
-166	- 40
-167	0
-168	30
-169	45
-170	60
-171	65
-172	70
-173	65
-174	60
-175	45
-176	20
-177	0
-178	- 30
-179	- 45
-180	- 60
-181	- 65
-182	- 70
-183	- 70
-184	- 60
-185	- 45
-186	- 25
-187	0
-188	60
-189	90
-190	110
-191	125
-192	130
-193	125
-194	115
-195	100
-196	80
-197	60
-198	30
-199	0
-200	- 20
-201	- 50
-202	- 80
-203	-110
-204	-125

Year	Wolf Number
-205	-130
-206	-125
-207	-100
-208	- 70
-209	- 30
-210	0
-211	30
-212	70
-213	120
-214	130
-215	120
-216	80
-217	50
-218	20
-219	0

Year	Wolf Number

APPENDIX B

Fortran IV Program for Prediction Functions

```
      INTEGER T
      DIMENSION XW(20)
C
      WRITE(5,2)
    2 FORMAT(1H1,'DATA')
      PI = 3.1416
      DO 75 J = 1300,2030,10
      L = 0
      T = J
      K=J+9
      DO 65 I=J,K
      WC1=0
      WC2=0
      WC3=0
      WC4=0
      WC5=0
      WC6=0
      WC7=0
      DO 15 N=1,24
   15 WC1 = WC1 + COS((12.+2*N)*PI*(I-1159)/399.)
      DO 20 N=1,8
   20 WC2 = WC2 + COS((30.+2*N)*PI*(I-1149)/399.)
      DO 25 N=1,8
   25 WC3 = WC3 + COS((37.+2*N)*PI*(I-1358)/500.)
      DO 35 N=1,20
   35 WC4 = WC4 + COS((38.+2*N)*PI*(I-1171)/607.)
      DO 40 N=1,12
      WC5 = WC5 + COS((41.+2*N)*PI*(I-1116)/610.)
   40 WC6 = WC6+COS((18.+2*N)*PI*(I-1249)/376.)
      DO 45 N=1,32
   45 WC7 = WC7+COS((26.+2*N)*PI*(I-730)/641.)
      WS1 = (2.0+SIN(80.*PI*(I-1159)/399.
      WS2 = -(2.0+SIN(80.*PI*(I-1149)/399.))
      WS3 =  (2.0+SIN(94.*PI*(I-1358)/500.))
      WS4 = (2.0+SIN(122.*PI*(I-1171)/607.))
      WS5 =  (2.0+SIN(110.*PI*(I-1116)/610.))
      WS6 = (2.0+SIN(64.*PI*(I-1249)/376.))
      WS7 = -(2.0+SIN(116.*PI*(I-730)/641.))
      W1=1.88*WC1*WS1
      W2= 4.75*WC2*WS2
      W3= 5.63*WC3*WS3
      W4= 2.00*WC4*WS4
      W5= 3.75*WC5*WS5
      W6= 3.33*WC6*WS6
      W7= 1.094*WC7*WS7
      L = L +1
      XW(L) = W1+W2+W3+W4+W5+W6+W7
   65 CONTINUE
      WRITE(5,90)T,(XW(L),L=1,10)
   75 CONTINUE
   90 FORMAT(1X,I4,10F9.1)
      CALL EXIT
      END
```

APPENDIX C

Calculated Annual Wolf Numbers (1300 A.D. - 2030 A. D.) Using Seven Prediction Functions.

Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number
1300	-31	1335	25	1370	-168
1301	-25	1336	45	1371	-183
1302	-21	1337	67	1372	-169
1303	-17	1338	76	1373	-128
1304	-14	1339	65	1374	-74
1305	-11	1340	39	1375	-24
1306	-7	1341	11	1376	12
1307	-1	1342	-9	1377	36
1308	7	1343	-16	1378	58
1309	14	1344	-19	1379	83
1310	20	1345	-28	1380	111
1311	23	1346	-47	1381	129
1312	25	1347	-74	1382	126
1313	27	1348	-98	1383	100
1314	32	1349	-106	1384	61
1315	41	1350	-96	1385	22
1316	50	1351	-71	1386	-4
1317	54	1352	-39	1387	-18
1318	48	1353	-10	1388	-28
1319	32	1354	12	1389	-41
1320	12	1355	30	1390	-57
1321	-4	1356	51	1391	-68
1322	-13	1357	80	1392	-66
1323	-16	1358	116	1393	-46
1324	-22	1359	148	1394	-18
1325	-36	1360	165	1395	8
1326	-57	1361	156	1396	23
1327	-73	1362	122	1397	25
1328	-72	1363	74	1398	23
1329	-53	1364	26	1399	23
1330	-22	1365	-11	1400	28
1331	4	1366	-38	1401	34
1332	17	1367	-64	1402	36
1333	17	1368	-96	1403	29
1334	16	1369	-135	1404	16

Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number
1405	2	1450	8	1495	-9
1406	-9	1451	12	1496	-14
1407	-14	1452	12	1497	-21
1408	-13	1453	12	1498	-23
1409	-11	1454	15	1499	-20
1410	-10	1455	19	1500	-13
1411	-10	1456	21	1501	-6
1412	-11	1457	17	1502	-4
1413	-13	1458	7	1503	-6
1414	-14	1459	-3	1504	-8
1415	-14	1460	-7	1505	-8
1416	-14	1461	-6	1506	-4
1417	-13	1462	-1	1507	-1
1418	-10	1463	2	1508	1
1419	-7	1464	0	1509	4
1420	-5	1465	-5	1510	9
1421	-6	1466	-9	1511	19
1422	-9	1467	-9	1512	29
1423	-13	1468	-5	1513	33
1424	-14	1469	-1	1514	31
1425	-12	1470	-1	1515	26
1426	-8	1471	-5	1516	25
1427	-4	1472	-10	1517	30
1428	-1	1473	-11	1518	37
1429	0	1474	-8	1519	37
1430	3	1475	-2	1520	26
1431	9	1476	2	1521	5
1432	19	1477	2	1522	-15
1433	28	1478	-3	1523	-27
1434	34	1479	-7	1524	-28
1435	35	1480	-7	1525	-24
1436	33	1481	-3	1526	-24
1437	28	1482	3	1527	-33
1438	22	1483	9	1528	-45
1439	15	1484	11	1529	-53
1440	5	1485	10	1530	-49
1441	-6	1486	10	1531	-34
1442	-17	1487	12	1532	-13
1443	-26	1488	13	1533	3
1444	-32	1489	13	1534	11
1445	-35	1490	9	1535	13
1446	-33	1491	2	1536	17
1447	-25	1492	-3	1537	30
1448	-14	1493	-6	1538	51
1449	-1	1494	-6	1539	70

Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number
1540	74	1585	12	1630	34
1541	56	1586	7	1631	12
1542	24	1587	1	1632	-4
1543	-7	1588	-4	1633	-14
1544	-26	1589	-6	1634	-21
1545	-34	1590	-7	1635	-29
1546	-44	1591	-8	1636	-40
1547	-70	1592	-11	1637	-51
1548	-110	1593	-15	1638	-59
1549	-144	1594	-16	1639	-60
1550	-149	1595	-14	1640	-54
1551	-116	1596	-7	1641	-43
1552	-60	1597	3	1642	-30
1553	-6	1598	13	1643	-17
1554	27	1599	22	1644	-8
1555	44	1600	29	1645	-3
1556	64	1601	37	1646	-3
1557	104	1602	44	1647	-6
1558	156	1603	50	1648	-8
1559	194	1604	49	1649	-8
1560	193	1605	40	1650	-4
1561	151	1606	25	1651	2
1562	88	1607	9	1652	7
1563	33	1608	-1	1653	9
1564	2	1609	-4	1654	8
1565	-13	1610	-4	1655	7
1566	-29	1611	-12	1656	8
1567	-57	1612	-31	1657	12
1568	-91	1613	-56	1658	21
1569	-112	1614	-78	1659	33
1570	-108	1615	-86	1660	45
1571	-83	1616	-77	1661	52
1572	-51	1617	-56	1662	51
1573	-27	1618	-34	1663	42
1574	-14	1619	-17	1664	29
1575	-9	1620	-7	1665	17
1576	-4	1621	3	1666	8
1577	3	1622	15	1667	1
1578	7	1623	34	1668	-7
1579	6	1624	58	1669	-17
1580	2	1625	79	1670	-28
1581	-2	1626	94	1671	-37
1582	0	1627	95	1672	-40
1583	6	1628	83	1673	-38
1584	11	1629	61	1674	-32

Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number
1675	-25	1720	6	1765	-19
1676	-17	1721	-23	1766	-21
1677	-8	1722	-35	1767	-31
1678	1	1723	-41	1768	-52
1679	8	1724	-50	1769	-74
1680	9	1725	-69	1770	-80
1681	3	1726	-92	1771	-62
1682	-6	1727	-106	1772	-27
1683	-14	1728	-98	1773	8
1684	-16	1729	-68	1774	28
1685	-13	1730	-27	1775	32
1686	-8	1731	8	1776	34
1687	-5	1732	28	1777	46
1688	-3	1733	34	1778	69
1689	-1	1734	35	1779	90
1690	6	1735	41	1780	91
1691	20	1736	55	1781	66
1692	38	1737	72	1782	24
1693	54	1738	82	1783	-15
1694	62	1739	79	1784	-37
1695	59	1740	62	1785	-41
1696	48	1741	37	1786	-41
1697	33	1742	12	1787	-48
1698	19	1743	-7	1788	-63
1699	5	1744	-16	1789	-77
1700	-10	1745	-18	1790	-75
1701	-27	1746	-23	1791	-55
1702	-47	1747	-37	1792	-23
1703	-68	1748	-58	1793	6
1704	-86	1749	-77	1794	24
1705	-94	1750	-84	1795	31
1706	-90	1751	-71	1796	33
1707	-73	1752	-43	1797	37
1708	-49	1753	-14	1798	42
1709	-22	1754	5	1799	43
1710	1	1755	9	1800	36
1711	20	1756	10	1801	23
1712	35	1757	19	1802	8
1713	52	1758	40	1803	-1
1714	75	1759	66	1804	-5
1715	98	1760	79	1805	-5
1716	113	1761	70	1806	-5
1717	110	1762	41	1807	-6
1718	86	1763	9	1808	-5
1719	46	1764	-12	1809	-2

Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number
1810	2	1855	-27	1900	-22
1811	5	1856	-40	1901	-30
1812	4	1857	-62	1902	-35
1813	-1	1858	-87	1903	-32
1814	-7	1859	-103	1904	-20
1815	-15	1860	-99	1905	-7
1816	-22	1861	-77	1906	0
1817	-28	1862	-44	1907	-2
1818	-31	1863	-13	1908	-8
1819	-29	1864	8	1909	-11
1820	-23	1865	20	1910	-3
1821	-14	1866	31	1911	10
1822	-4	1867	48	1912	23
1823	6	1868	72	1913	28
1824	17	1869	96	1914	26
1825	30	1870	107	1915	23
1826	45	1871	98	1916	26
1827	57	1872	71	1917	34
1828	60	1873	36	1918	39
1829	50	1874	5	1919	34
1830	31	1875	-16	1920	19
1831	9	1876	-30	1921	2
1832	-7	1877	-46	1922	-9
1833	-17	1878	-69	1923	-11
1834	-24	1879	-96	1924	-10
1835	-38	1880	-115	1925	-13
1836	-59	1881	-117	1926	-26
1837	-82	1882	-97	1927	-43
1838	-96	1883	-62	1928	-55
1839	-91	1884	-26	1929	-55
1840	-65	1885	3	1930	-40
1841	-30	1886	22	1931	-21
1842	0	1887	35	1932	-7
1843	19	1888	48	1933	-4
1844	27	1889	63	1934	-7
1845	35	1890	76	1935	-5
1846	54	1891	80	1936	11
1847	82	1892	72	1937	39
1848	108	1893	52	1938	66
1849	117	1894	26	1939	75
1850	100	1895	3	1940	61
1851	66	1896	-12	1941	33
1852	27	1897	-17	1942	9
1853	-2	1898	-16	1943	-1
1854	-18	1899	-16	1944	-1

Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number	Year	Calc. Annual Wolf Number
1945	-10	1990	-88		
1946	-42	1991	-91		
1947	-94	1992	-79		
1948	-141	1993	-54		
1949	-152	1994	-22		
1950	-121	1995	10		
1951	-65	1996	36		
1952	-15	1997	53		
1953	8	1998	67		
1954	12	1999	84		
1955	25	2000	109		
1956	67	2001	139		
1957	132	2002	162		
1958	187	2003	164		
1959	201	2004	136		
1960	164	2005	85		
1961	98	2006	28		
1962	38	2007	-15		
1963	4	2008	-35		
1964	-8	2009	-42		
1965	-19	2010	-53		
1966	-43	2011	-80		
1967	-79	2012	-119		
1968	-107	2013	-152		
1969	-109	2014	-160		
1970	-85	2015	-139		
1971	-49	2016	-96		
1972	-18	2017	-52		
1973	-1	2018	-18		
1974	5	2019	3		
1975	10	2020	17		
1976	19	2021	32		
1977	31	2022	49		
1978	39	2023	64		
1979	37	2024	71		
1980	25	2025	69		
1981	9	2026	60		
1982	-6	2027	46		
1983	-17	2028	30		
1984	-21	2029	14		
1985	-24	2030	0		
1986	-28				
1987	-39				
1988	-55				
1989	-74				

PROGRESS REPORT - SUNSPOTS PREDICTION METHODS

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The work at the Graduate Institute of Technology started out with an intensive search of literature for any mention of sunspots and their nature. All the available Wolf numbers were plotted and several attempts were made to smooth these data. The thirteen month moving average was used because of its historic importance but a thirty-one month scheme smoothed some of the noise better than other periods of time. This smoothing was done considering the original data as positive one cycle and negative the next as well as the usual method of considering all data as positive. In addition to this, work is in progress trying to find a prediction method that offers a better result than any we have found to date.

Efforts too numerous to enumerate in detail but well documented by Rubashev and Vitinskii discuss the sunspot phenomena and attempts at its prediction of future activity. The attempts at prediction are based on the Wolf numbers which are considered an accurate indication of sunspot activity going back to 1750. Attempts at prediction have been mainly concentrated on assuming a certain functional behavior for a cycle or a group of cycles with certain parameters contained in the functional expressions that are determined by empiro-statistical means. Most of these attempts are summarized by Vitinskii. An attempt is in progress that applies Wiener's weighted moving averages. This procedure is usually applied to discrete quantities, and for the prediction of the next sunspot cycle would be applied to discrete quantities determined for each cycle such as the maximum Wolf number, the rise time, the fall time, and possibly other determinable quantities. The reason for the choice of this procedure by the team at the University of Arkansas is documented in another section of this report.

In the literature survey connected with the task of predicting the next sunspot cycle, the team at the University of Arkansas ran across correlated material on sunspots, their behavior in general, possible causes, and discussion of stellar-solar-terrestial relationships in general by Ellsworth Huntington that may prove of considerable value in sunspot predicting. It is surprising that these efforts by Huntington, mostly accomplished before 1925, were not mentioned by such writers on solar activity as Kiepenheuer, de Jager, Rubashev, and Vitinskii. Even Stetson in his book written for the layman mentions Huntington in connection with his studies of the effect of climate on man, but does not mention the efforts of Huntington concerned directly with sunspots.

The efforts of Huntington directly related to sunspots are found principally in two books, "Earth and Sun" and "Climate Changes". Additional related information is found in two other books, "The Climatic Factor" and "Civilization and Climate". Perhaps an explanation, somewhat subjective in nature, is now in order concerning the lack of knowledge about Huntington's activities and writings about sunspots and their stellar-solar-terrestial relationship.

In the first place Huntington is noted as an explorer and geographer with voluminous writings in these areas. Though his books on geography had a wide circulation, his books in related areas had a small distribution. In geography Huntington was a strong proponent of environmentalism. An example of this is his subscription to the thesis that the fall of the Greek and Roman civilization was brought about by a change in climate that permitted the malaria mosquito to flourish with the resultant detrimental effect on the health of the Greeks and Romans. This view is, of course, refuted by such present day antienvironmentalists as Toynbee. The effect of the switch away from environmentalism during the past generation has greatly decreased the interest in Huntington's work. In fact, the book "Climate Changes" in the University of Arkansas library was checked out only in 1941, 1954, 1957, 1967.

Another criticism that a present-day scientist will make when he reads Huntington's efforts in other than geography is that Huntington draws rather far-fetched conclusions from a limited number of facts, and his correlation of data leaves something to be desired. However, this was quite common among his contemporaries and any contribution of his that is of value in present-day scientific endeavors should not be discarded because of this propensity.

Another reason that modern scientists may be reluctant to investigate the efforts of Huntington in the area of sunspots is that he mentions as a possible cause an electro-magnetic hypothesis. In his scientism Velikovsky accepts this hypothesis as a scientific fact, and expounds on it with religious fervor. As is well known, Velikovsky accepts as facts certain other far-out hypotheses, lists data that supports these hypotheses, and ignores data that contradict these hypotheses. The result has been that many modern scientists, being human, have thr-- up a mental block against any hypothesis expounded by Velisovsky. The fact of the matter is that Velikovsky may have indicated that certain hypothetical facts originated with him, when in reality he should have given credit to earlier writers who posed these same ideas as rather far fetched hypotheses.

One of these appears to be the concept of an electro-magnetic influence that celestial bodies supposedly have on each other. An example of the contract between the scientific approach and the approach influenced by scientism will now be given by quoting both Velikovsky and Huntington on the subject of the electro-magnetic hypothesis as related to celestial bodies.

Velikovsky writes: "The electromagnetic nature of the universe, deduced in Worlds in Collision from a series of historical phenomena, is supported by another series of recent observations." Velikovsky then lists what he considers supporting evidence from certain sources, one of which is the New York Times. Velikovsky then continues: "These are only a few of the recent discoveries that make a revision of the mechanistic concept of the universe quite mandatory. Exactly because of the accuracy achieved without reckoning with forces that appear to exist, celestial mechanics, a solid work of great mathematical minds for almost three centuries, may seem even more in need of such revision. All this has little direct bearing on the story of Worlds in Collision, which claims only the effect to be expected if a magnetic body like the earth should come very close to another magnetic body. It was my skepticism concerning the infallibility of the celestial mechanics, which assumes the celectial bodies to be electrically and magnetically sterile, that was the real cause of the emotional outburst.

"Let us think of a binary or double star; both stars revolve around each other or a common center. A half-revolution period of a few days or only hours is common. Let us assume that the stars of the binary are magnets 7000 gauss strong. It is immediately obvious that even should the electrical component of the electromagnets be disregarded these stars are not moving in a system purely mechanical."
The writer will let the reader draw his own conclusions concerning the scientific verity of certain statements made by Velikovsky in the preceding quote.

Contrast this with a statement made by Huntington about thirty years earlier on the same topic. "The agency through which the planets influence the solar atmosphere is not yet clear. The suggested agencies are the direct pull of gravitation, the tidal effect of the planets, and an electro-magnetic effect. In Earth and Sun the conclusion is reached that the first two are out of the question, a conclusion in which E. W. Brown acquiesces. Unless some unknown cause is appealed to, this leaves an electro-magnetic hypothesis as the only one which has a reasonable foundation. Schuster inclines to this view. The conclusions set forth in Earth and Sun as to the electrical nature of the sun's influence on the earth point somewhat in the same direction. Hence, in this chapter we shall inquire what would happen to the sun, and hence to the earth, on their journey through space, if the solar atmosphere is actually subject to disturbance by the electrical or other effects of other heavenly bodies. It need hardly be pointed out that we are here venturing into highly speculative ground, and that the verity or falsity of the conclusions reached in this chapter has nothing to do with the validity of the reasoning in previous chapters. Those chapters are based on the assumption that terrestial causes of climatic changes are supplemented by solar disturbances which produce their effect partly through variations in the intensity and paths of cyclonic storms. The present chapter seeks to shed some light on the possible causes and sequences of solar disturbances." Huntington's comment on a double star will be quoted later.

When a person unrolls and looks at a plot of the available smoothed Wolf numbers (whether they have been smoothed according to a yearly pattern or a pattern of greater length such as thirty-one months) the so-called eleven year periodicity is immediately evident and an eighty to ninety year period can also be readily seen. However, one is also struck by the fact that the nineteenth, or last complete eleven year cycle exhibited the highest Wolf numbers ever recorded since their inception around 1750. Again looking at the plot of the smoothed Wolf numbers and trying to predict the behavior of the twentieth cycle, a person can validly draw three conclusions:

a. That the twentieth cycle maximum will show a marked drop from the nineteenth cycle such as occurred from the eleventh to the twelfth cycle.

b. That the twentieth cycle maximum will be approximately equal to the nineteenth as occurred from the third to the fourth cycle.

c. That the twentieth cycle maximum will be larger than the nineteenth. This would postulate a cycle of greater length than the time period for which Wolf number data is available. A cycle of this type of 600 years is postulated by such authors as Ruboshev.

Statistical methods can be used, and have been used in profusion, to predict the statistical probability that the event described in (a) or the one described in (b) occurs. These predictions have been summarized by Vitinskii. In these predictions, the forecasts for the maximum of the Wolf number during the twentieth cycle range from 44 to 160 and the prediction of the epoch of the maximum ranges from 1968 to 1972.5.

These results lead Vitinskii to the following conclusions: "We have considered the principal empiricostatistical methods for the long-range forecasting of Wolf numbers and we have shown that the reliability of the results obtained using these methods still leave much to be desired. What, then, are the possible ways in which solar-activity forecasts can be developed in the future?

"First, a comprehensive approach to this problem must be worked out. Even the most perfect theory of solar activity, if it were developed in the near future, would not give completely reliable results. However, at present we still do not have anything which remotely resembles such a theory, so that as a first stage we should seek some way in which satisfactory results can be obtained even using just the individual theoretical conclusions of solar physics."

Such comments as the preceding one show that very little progress has been made in the last half century in the area of solar activity forecasts. It is for this reason that the efforts of Huntington may be of considerable value in developing an improved procedure of solar-activity forecasting. His efforts are much more interdisciplinary than those of present day researchers whose efforts tend to become highly departmentalized. In the following paragraphs certain findings and opinions of Huntington which may be of value in determining a more accurate procedure for solar-activity forecasting will be discussed.

The first question that is always asked is what causes sunspot activity. Is its cause endogenic or exogenic? Modern workers in the area have tended toward the endogenic concept while earlier workers concentrated their efforts correlating exogenic causes, such as the tidal influence of the planets, with sunspot activity. Rubashev has a rather low opinion of these efforts as shown by the following quote: "We have discussed this problem in considerable detail because investigations of the tidal influence of the planets on the sun are still appearing, such as a number of papers by Link. These studies have a purely empirical-statistical character and warrant the criticism given them. Theoretical estimates of the possible effect are, in our opinion, far more useful than the numerous, but unpromising empirical attempts to detect such tidal influence."

Here is what Huntington wrote about the cause of sunspots more than forty years ago. "It is generally assumed that sunspots, solar prominences, the bright clouds known as faculae, and other phenomena denoting a perturbed state of the solar atmosphere, are due to some cause within the sun. Yet the limitation of these phenomena, especially the sunspots, to restricted latitudes, as has been shown in Earth and Sun, does not seem to be in harmony with an internal solar origin, even though a banded arrangement may be normal for a rotating globe. The fairly regular periodicity of the sunspots seems equally out of harmony with an internal origin. Again the solar atmosphere has two kinds of circulation, one the so-called "rice grains", and the other the spots and their attendant phenomena.

Now the rice grains present the appearance that would be expected in an atmospheric circulation arising from the loss of heat by the outer parts of a gaseous body like the sun. For these reasons and others, numerous good thinkers from Wolf to Schuster have held that sunspots owe their periodicity to causes outside the sun. The only possible cause seems to be the planets, acting either through gravitation, through forces of an electrical origin, or through some other agency. Various new investigations which are described in Earth and Sun support this conclusion. The chief difficulty in accepting it hitherto has been that although Jupiter, because of its size, would be expected to dominate the sunspot cycle, its period of 11.86 years has not been detected. The sunspot cycle has appeared to average 11.2 years in length, and has been called the 11-year cycle. Nevertheless, a new analysis of the sunspot data shows that when attention is concentrated upon the major maxima, which are least subject to retardation or acceleration by other causes, a periodicity closely approaching that of Jupiter is evident. Moreover, when the effects of Jupiter, Saturn and other planets are combined, they produce a highly variable curve which has an extraordinary resemblance to the sunspot curve. The method by which the planets influence the sun's atmosphere is still open to question. It may be through tides, through the direct effect of gravitation, through electro-magnetic forces, or in some other way. A detailed description of the planetary hypothesis of the 11-year sunspot period is found in Chapters X to XIII in Earth and Sun. A figure from this book is attached to this report.

However, Huntington's most startling prediction is associated with the so-called 80 to 90 year cycle. Here is what he writes concerning the double star Alpha Centauri. "If Alpha Centauri is really so important, the effect of its variations, provided it has any, ought perhaps to be evident in the sun. The activity of the star's atmosphere presumably varies, for the orbits of the two components have an eccentricity of 0.51. Hence, during their period of revolution, 81.2 years, the distance between them ranges from 1,100,000,000 to 3,300,000,000 miles. They were at a minimum distance in 1388, 1459, 1550, 1631, 1713, 1794, 1875, and will be again in 1956. In Fig. 11 showing sunspot variations, it is noticeable that the years 1794 and 1875 come just at the ends of periods of unusual solar activity, as indicated by the heavy horizontal line. A similar period of great activity seems to have begun about 1914. If its duration equals the average of its two predecessors, it will end about 1950. (Here he seems to contradict himself as the year 1957 is given in the figure). Back in the fourteenth century a period of excessive solar activity, which has already been described, culminated from 1370 to 1385, or just before the two parts of Alpha Centauri were at a minimum distance. Thus in three and perhaps four cases the sun has been unusually active during a time when the two parts of the star were most rapidly approaching each other and when their atmospheres were presumably most disturbed and their electrical emanations strongest." It should be remembered that this prediction was made in a book published in 1922. This poses the following question. Is there a relationship between the minimum distance year 1956 and the sunspot maximum occurring in 1957, or is this just a fortuitous coincidence?

In connection with longer sunspot cycles Huntington writes the following about the middle of the fourteenth century during which, according to Schove, the sunspots may have been as numerous as the present era. "Let us begin by scanning the available evidence as to solar disturbances previous to the time when accurate sunspot records are available. Two rather slender bits of evidence point to cycles of solar activity lasting hundreds of years. One of these has already been discussed in Chapter VI, where the climatic stress of the fourteenth century was described. At that time sunspots are known to have been unusually numerous, and there were great climatic extremes. Lakes overflowed in Central Asia; storms, droughts, floods, and cold winters were unuaually severe in Europe; the Caspian Sea rose with great rapidity; the trees in California grew with a vigor unknown for centuries; the most terrible of recorded famines occurred in England and India; the Eskimos were probably driven south by increasing snowiness in Greenland; and the Mayas of Yucatan appear to have made their last weak attempt at a revival of civilization under the stimulus of greater storminess and less constant rainfall." This brings up several questions. Just how valid is the correlation between climate extremes and sunspot activity? Is there a 600-year cycle as postulated by Rubashev? If there is a correlation between climate extremes and sunspot activity and if the 600-year cycle exists, does the recent unprecedented flood in Florence, Italy, and the snowfall in Mexico City, Mexico portend the end of a sunspot cycle extreme or does it portend still greater sunspot activity during the next cycle?

It is hoped that an evaluation of interdisciplinary efforts by Huntington related to sunspots will assist in the determination of the proper weighing factors to be used in the contemplated effort involving the Weiner Weighted Moving Averages.

In our work with mathematical prediction, it soon became apparent that all prediction depends in one form or another with the knowledge of a weighing function. Weiner says that his method of prediction is linear, invariant with respect to the choice of an origin in time, and dependent only on the past and present of the function under investigation. He puts most of his work into continuous time series and discusses discrete time series as an afterthought.

A weighing function is an absolute necessity. Blackman treats the case of a polynomial weighing function, Weiner uses any continuous function that has a Fourier transform. If we use the discrete data, we can go to Z transforms but if the infinite series cannot be turned into a closed form by summation or otherwise this is of little value.

We have tried several functions to approximate a single cycle. If an adequate function can be found and fitted to successive cycles, the variation of its parameters might be something that could be predicted using some scheme similar to the McNish-Lincoln method. We have not gone to the big computer yet because we still hope to find a better function to fit to each cycle.

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